DM 2015

V International Workshop on Direct Methods

Oxford, England,

6^{th}-8^{th} September 2015

DM 2015

V International Workshop on Direct Methods
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List of Delegates

Barbera, Daniele	University of Strathclyde
Barrera, Olga	Oxford University
Bleyer, Jérémy	Université Paris-Est
Boulbibane, Mostapha	University of Bolton
Charkaluk, Eric	Ecole Centrale de Lille University
Chen, Geng	RWTH Aachen University
Cocks, Alan	Oxford University
Fuschi, Paolo	Universita Mediterranea di Reggio Calabria
Fuessl, Josef	Vienna University of Technology
François, Stijn	Ku Leuven University
Gordon, Jerard	Lehigh University
Harris, David	Manchester University
Liu, Yinghua	Tsinghua University
Li, Mingjing	Vienna University of Technology
Leonetti, Leonardo	University of Calabria
Liu, Shu	University of Nottingham
Muñoz, Jose	Universitat Politecnica de Catalunya
Magisano, Domenico	University of Calabria
Martin, Chris	Oxford University
Trình Trần, Ngọc	Aachen University of Applied Sciences
Peigney, Michael	Université Paris-Est
Ponter, Alan	Leicester University
Pisano, Aurora	Universita Mediterranea di Reggio Calabria
Smith, Colin	University of Sheffield
Spiliopoulos, Konstantinos	National Technical University of Athens
Staat, Manfred	Aachen University of Applied Sciences
Soner, Ismail	Lehigh University
Tereshin, Denis	South Ural State University
Vermaak, Natasha	Lehigh University
Wang, Juan	Nottingham University

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Agenda

Monday 7th September 2015

Session 1

Chair: Alan Cocks

09:00-09:05 Welcome & Introduction

09:05-09:45 Abstract 1 **Alan R. S. Ponter**

Shakedown and Limit States for a Class of Yield Conditions with a Non-associated Flow Rule

09:45-10.25 Abstract 2 **David Harris**

Extremum principles and a class of flows of granular materials

10:25-11:05 Abstract 3 **Stijn François**

The performance of 3D volume elements in upper bound limit analysis: application to failure criteria of porous materials – Stijn François, Jérémy Bleyer, Eric Lemarchand, Luc Dormieux

11:05-11:20 Coffee

Session 2

Chair: Natasha Vermaak

11:20-12:00 Abstract 4 **Juan Wang**

Recent progress on lower-bound shakedown analysis of road pavements, Juan Wang, Shu Liu, Hai-Sui Yu

12:00-12:40 Abstract 5 **Colin Smith**

Recent advances in discontinuity layout optimization (DLO)

12:40-13:40 Lunch

Session 3

Chair: Eric Charkaluk

13:40-14:20 Abstract 6 Kostas V. Spiliopoulis

Shakedown solutions by the RSDM-S in a multiple loading domain, K,V. Spiliopoulos, K.D. Panagiotou

14:20-15:00 Abstract 7 Jose J. Muñoz

R-adaptivity in limit analysis

15:00-15:20 Tea

Session 4

Chair: Konstantinos Spiliopoulos

15:20-16:00 Abstract 8 Jérémy Bleyer

Numerical yield design analysis of high-rise reinforced concrete walls in fire conditions, Jérémy Bleyer, Duc Toan Pham, Patrick de Buhan

16:00-16:40 Abstract 9 Leonardo Leonetti

Shakedown analysis of 3D frames for complex statical & seismic load combinations, Leonardo Leonetti, Raffaele Casciaro, Giovanni Garcea

16:40-17:20 Abstract 10 Geng Chen

Study of effective strengths of particulate reinforced metal matrix composites (PRMMCs) using direct method and statistical learning, Geng Chen and Christoph Broeckmann and Dieter Weichert

17:20-18:00 Discussion

19:00-21:00 Dinner at St Anne's College

Tuesday 8th September 2015

Session 5

Chair: Paolo Fuschi

09:00-09:40 Abstract 11 Manfred Staat & Ngọc Trình Trần

Shakedown analysis under stochastic uncertainty by chance constrained programming, Ngọc Trình, Thanh Ngọc, H.G. Matthies, G.E. Stavroulakis & M. Staat

09:40-10.20 Abstract 12 Denis A. Tereshin

A direct solution technique and bounds for the steady state response of elastic-plastic structures on the basis of a discrete primal-dual formulation, Denis A. Tereshin

10:20-11:00 Abstract 13 Daniele Barbera

Micromechanical modelling on plastic and creep behaviours of MMCs using Linear Matching Method, Daniele Barbera, Dario Giugliano, Haofeng Chen, Yinghua Liu

11:00-11:20 Coffee

Session 6

Chair: Alan Ponter

11:20-12:00 Abstract 14 Eric Charkaluk

Shakedown state in polycrystals: a direct numerical assessment, Domenico Magisano, Eric Charkaluk, Gery de Saxce, Toufik Kanit

12:00-12:40 Abstract 15 Mingjing Li

Effective strength of wood cells determined by means of numberical limit analysis, M. Li, J. Füssl, M. Lukacevic, J. Eberhardsteiner

12:40-13:20 Abstract 16 Yinghua Liu

Lower Bound Limit Analysis of defective pipelines under combined loadings, Yinghua Liua, Bingye Xu and Xianhe Du

13:20-14:20 Lunch

14:20-16:00 Discussion and Conclusion

Shakedown and Limit States for a Class of Yield Conditions with a Non-associated Flow Rule

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Abstract

The paper will describe an approach to plasticity theorems for non-associated flow rules by studying an important class of problems where a convex yield condition depends on the hydrostatic pressure but the flow rule involves volume conserving plastic flow. There are two groups of problems that fall into this category. The first is the analogous problem of frictional contact between elastic bodies and the second is the deformation of geotechnical materials exhibiting Coulomb frictional behaviour. In particular the following plasticity model is discussed:

Yield condition: $$f = g(\sigma'_{ij}) - c - |p|\tan\phi\,, \quad p = \tfrac{1}{3}\sigma_{kk} \le 0\,, \quad \sigma'_{ij} = \sigma_{ij} - p\delta_{ij} \qquad (1)$$

Non associated flow rule $$\dot{\varepsilon}^{p}_{ij} = \frac{\partial g}{\partial \sigma'_{ij}}\,, \quad \dot{\varepsilon}^{p}_{kk} = 0 \qquad (2)$$

Here c is a cohesion and ϕ is a friction angle. For the case $c = 0$, purely frictional behaviour occurs and such yield conditions relate to the behaviour of granular materials.

The two classes of problem are discussed in sequence. Upper and lower bound shakedown theorems are derived which are linked by the distribution of initial slip or plastic strain, i.e. the assumed conditions at the beginning of the cycle of loading. The kinematic bound may be interpreted as an absolute bound on a suitably defined shakedown limit whereas the static bound is a bound on the kinematic bound for the same assumed initial conditions. This leads to the definition of consistent and non-consistent initial conditions. Consistent initial conditions lead to the definition of a cyclic solution corresponding to a shakedown boundary whereas non-consistent initial conditions do not lead to a shakedown limit.

The theory implies that more than one shakedown limit may exist corresponding, for example, by mechanisms occurring in distinct locations with differing initial conditions. However an extreme shakedown condition may be defined. This shakedown condition may be formulated as the solution of a kinematic programming problem where the variables are the initial distribution of plastic strain or slip and mechanisms of reverse plasticity of ratchetting. This problem is solved for a simple frictional slip problem by a method based upon the Linear Matching Method. The solution demonstrates that a shakedown limit may be defined within the shakedown limit defined by a classical Melan limit. This implies that there will be ranges of loading conditions where shakedown will certainly occur, may occur and certainly will not occur.

For the geotechnical problem, the limit load problem is discussed in detail. The theory allows new insight into traditional geotechnical methods and helps to interpret recent numerical solutions in the literature

To the author's knowledge, this is the first time a consistent set of shakedown and limit theorems have been developed for a non-associated flow rule.

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ARSP
April 2015

Extremum principles and a class of flows of granular materials

De Josselin de Jong [1,2] has pointed out that safety factors calculated using a classical plasticity model may be in serious error due to the fact that granular materials have additional degrees of kinematic freedom to those considered in the model, namely the freedom of the grains to rotate. Neglect of grain rotation is one of a number of deficiencies in classical models. For example, both the non-associated flow rule and the double shearing model (Spencer [3]) are ill-posed, when considered as a system of partial differential equations, for evolutionary problems.

A model is presented here, see Harris [4,5] which augments the plastic potential term in the expression for the plastic strain-rates by a term which is analogous to the non-coaxial term in the double-shearing model but which replaces the spin of principal axes of stress by a quantity present in a Cosserat continuum, namely the intrinsic spin. The equations governing the model are presented and explained.

The starting point for the consideration and calculation of design safety factors is the development of extremum principles for the model. The natural setting for the proof of extremum principles in plasticity theory is an associated flow rule. However, associated flow rules in the context of perfect plasticity do not provide a good model for granular materials, where pressure dependence of yield coupled with shear induced dilatancy are a dominant feature of behaviour. The classical way to resolve this problem is the use of a so-called non-associated flow rule but the classical proof of extremum principles does not generalise to this model. There have been attempts to develop extremum principles for non-associated flow rules but the results are much weaker than the corresponding results for an associated flow rule. Attempts to develop extremum principles for the double-shearing model have also failed.

The model presented here is closely related to both the plastic potential model and the double-shearing model and hence it seems that a proof of a useful extremum principle for a general flow is unlikely to exist. In this talk we consider a special class of flows (namely anti-plane strain - sometimes called rectilinear - flow) and demonstrate that extremum principles do exist for this special class of flows. The work is a generalisation of Spencer and O'Mahony, see [6], and O'Mahony [7].

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[7] T.C. O'Mahony, PhD thesis, University of Nottingham 1980

The performance of 3D volume elements in upper bound limit analysis: application to failure criteria of porous materials

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In this presentation, the performance of various element types for three-dimensional (3D) upper bound limit analysis (LA) is studied in view of the computation of failure criteria for porous materials. In general, the objective of LA is the determination of the ultimate limit state of structures exhibiting rigid-plastic material behavior. For a two-dimensional analysis, two element types are commonly used: (a) discontinuous linear velocity elements and (b) quadratic velocity simplex strain elements. Quadratic elements have been demonstrated to provide a more accurate alternative to the commonly used discontinuous linear velocity elements. In a 3D analysis, applications of quadratic fields are scarce but provide a promising alternative to improve accuracy and reduce computational costs.

As a first 3D benchmark problem, the isotropic compression of the hollow Gurson's sphere with a Drucker-Prager matrix is considered. It is demonstrated that the use of a continuous quadratic velocity field results in locking in the incompressible case (Von Mises). This is attributed to the imposed velocity on the outside surface of the Gurson sphere that is incompatible with the incompressiblity constraint. A discontinuous linear velocity field does not result in locking, but suffers from a slow convergence. Therefore, a third case is considered, allowing for kinematically admissible discontinuities between quadratic velocity elements. The discontinuities mitigate the locking phenomenon whereas the quadratic velocity field results in a fast convergence.

The discontinuous quadratic approach is further employed to compute failure criteria for pressure-sensitive porous materials, both on the Gurson sphere and a periodic RVE. For the Gurson sphere, a significant J3 effect is observed, while the failure surface is generally over-estimated with respect to the periodic RVE where strain localization results in a local failure.

Recent progress on lower-bound shakedown analysis of road pavements

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Abstract:

Shakedown theory has been recognised as a more rational basis for structural design of flexible road pavements. A lower-bound shakedown approach, which aims to find the maximum design load of pavement structures, was developed by the Nottingham Centre for Geomechanics (NCG) in the University of Nottingham (Yu 2005, Wang 2011, Yu and Wang 2012, Wang and Yu 2013a), that forms part of efforts among other researchers' (e.g. Ponter et al. 1985, Yu and Hossain 1998; Li and Yu 2006, Nguyen et al. 2008) in applying shakedown theory in pavement designs. The lower-bound shakedown solutions were consistent well with the shakedown limits obtained by a numerical step-by-step approach (Wang and Yu 2013b, Liu et al. 2014) assuming that the materials are isotropic and homogeneous following associated plastic rule. Recently, this lower-bound approach was further developed considering more realistic cases. Both two-dimensional and three-dimensional shakedown analyses were carried out taking into account cross-anisotropic (Wang and Yu 2014; Yu et al. 2015) or inhomogeneous materials, the properties of which were programmed into a finite element software ABAQUS. For pavement materials obeying non-associated flow rule, the corresponding two-dimensional lower-bound shakedown limits were also estimated by extending the lower-bound shakedown approach. The numerical step-by-step approach was also applied to address the non-associated problems and obtained similar results. Through these studies, influences of the original assumptions on the shakedown-based pavement designs can be finally assessed.

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Recent advances in discontinuity layout optimization (DLO)

In this paper recent advances in the development of the DLO methodology are described, and results for a range of benchmark problems are presented. Advances described include: (i) modelling rotational failure mechanisms in restrictive geometries and including frictional materials; (ii) modelling concrete slab stability; (iii) geometry optimization.

The first issue will be explored in detail, while more illustrative examples of the latter two issues will be given. Following a brief overview of the modelling of rotational failure in DLO, the issue of how the yield surface for a rotational discontinuity is modified by a restrictive boundary is explored together with how discontinuities can be combined. Examples such as Fig 1 show how the classic Taylor slope stability problem involves a somewhat more complex mechanism than a single curved slipline, (though the load factor is only marginally changed). The geometrical and LP formulation issues for frictional materials are then explored showing how friction brings in more complexity, but at the same time retains an elegance in the formulation. Example solutions to weightless cohesive-frictional problems will be presented.

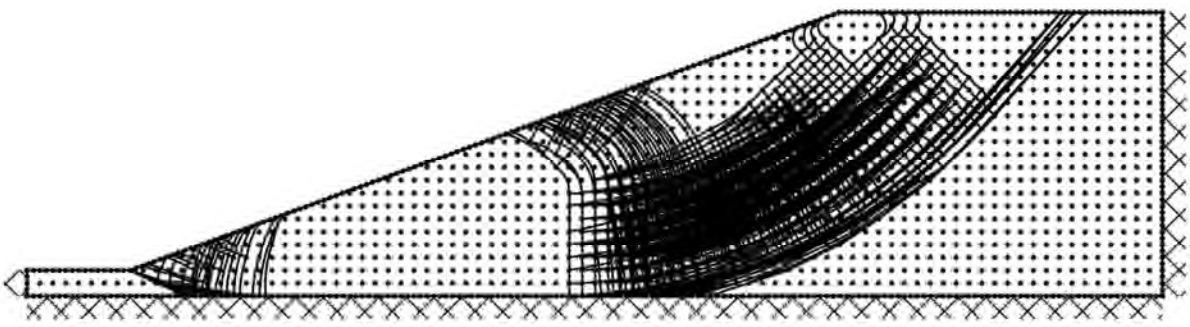

Figure 1: Taylor slope stability problem

SHAKEDOWN SOLUTIONS BY THE RSDM-S IN A MULTIPLE LOADING DOMAIN

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Abstract

One of the major concerns of a structural engineer is to determine whether a cyclic loading, applied on a structure or a structural component, causes plastic deformations that may lead the structure to undesired failures, like low cycle fatigue or ratcheting. These types of the long term structural response are avoided when the loading intensity is below elastic shakedown. A method that may predict this response, without following cumbersome time stepping calculations, is the Direct Method called RSDM (Residual Stress Decomposition Method) [1].

If, on the other hand, safety margins are sought so that the long term response is elastic, a Direct Method that has been named Residual Stress Decomposition Method for Shakedown (RSDM-S) appeared recently in the literature [2]. The method may provide the shakedown load factor for cyclic mechanical [2] or thermomechanical load [3]. The approach has a common origin with the RSDM since they both exploit the expected cyclic nature of the residual stresses at the cyclic state. Thus, the unknown residual stresses are decomposed into Fourier series. The Fourier coefficients in the RSDM-S are evaluated in an iterative way, while shrinking the load domain through a systematic reduction of the shakedown load factor. Iterations stop when the only remaining terms are the constant terms in the series. A perfectly plastic material with a von Mises yield surface is assumed. The approach is formulated within the framework of the finite element method. The stiffness matrix needs to be formed and decomposed only once. The whole approach is shown to be stable and computationally efficient, with uniform convergence. One of the main advantages of the approach is that there is no need to use mathematical programming algorithms and thus it may be amended in any finite element program.

In the present work main computational and convergence issues of the RSDM-S are discussed. Also, a tolerance criterion, that improves the computational time, is adopted. The method, which until now was applied to structures subjected to two loads, is herein extended to thermomechanical loadings that consist of three loads. Further extension of the method to multiple loading domains is also discussed.

Examples of application for various structures and loadings will be presented during the workshop.

References

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R-adaptivity in limit analysis

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1 Introduction

The goal is to include the nodal positions as an additional optimisation variable in the standard upper and lower bound formulations in limit analysis. This is a similar idea to the perturbation analysis in upper bound formulations with rigid blocks introduced in [2], which we here extend to more general finite elements in limit analysis [5, 6, 4, 8].

This work is related to similar strategies where the nodal positions of the problem at hand are optimised in order to improve the accuracy of the resuls. This type of anlysis has been previously adopted in elasticity [9], elastodynamics [10], or biomechanics [3, 7].

2 Preliminary background

Upper and lower bound solutions may be written as a second-order conic programming (SOCP) that has the following general form,

$$\begin{aligned} \textbf{Primal: } \lambda^* = \max_{\lambda,\boldsymbol{\sigma}} \lambda & \\ s.t.\ \mathbf{X}\boldsymbol{\sigma} + \lambda\mathbf{f} = \mathbf{b} & \\ \boldsymbol{\sigma} \in \mathcal{K} & \end{aligned} \tag{1}$$

Here, the global vector $\boldsymbol{\sigma}$ denotes stress variables, which have been conveniently transformed in order to write the plastic criteria in the form $\boldsymbol{\sigma} \in \mathcal{K}$, with $\mathcal{K}$ a second-order cone. The variable λ is the load factor, which is maximised inorder to compute the bearing capacity of the problem at hand.

The matrix $\mathbf{X}$ and vector $\mathbf{f}$ depend on the discretisation of the domain, that is, on the nodal positions $\mathbf{x}$ and the triangularisation $\mathcal{T}$ employed. If these are considered fixed, as it is usually the case, and for some common plastic criteria, the problem in (1) is convex. The lower and upper formulations of limit analysis require different forms of the matrix $\mathbf{X}$ and vector $\mathbf{b}$, which may be found elsewhere [5, 6, 4, 8].

The problem in (1) is the standard form used for the lower bound (LB) limit analysis. The upper bound (UB) problem is generally written in the dual form of this problem, which physically corresponds to minimisation of the dissipation power. It will become convenient to derive next this dual form.

The Lagrangian function of the problem in (1) reads [1]:

$$\mathcal{L}(\boldsymbol{\sigma}, \lambda; \mathbf{v}, \boldsymbol{\omega}) = \lambda + \mathbf{v}^T(\mathbf{b} - \mathbf{X}\boldsymbol{\sigma} - \lambda\mathbf{f}) - \boldsymbol{\omega}^T\boldsymbol{\sigma} \tag{2}$$

The optimal value λ^* may be then obtained as

$$\lambda^* = \max_{\boldsymbol{\sigma},\lambda} \min_{\boldsymbol{\omega}\in\mathcal{K}^*,\mathbf{v}} \mathcal{L}(\boldsymbol{\sigma}, \lambda; \mathbf{v}, \boldsymbol{\omega}) = \min_{\boldsymbol{\omega}\in\mathcal{K}^*,\mathbf{v}} \max_{\boldsymbol{\sigma},\lambda} \mathcal{L}(\boldsymbol{\sigma}, \lambda; \mathbf{v}, \boldsymbol{\omega}) \tag{3}$$

where the second equality holds due to strong duality. In general, the dual $\mathcal{S}^*$ of a set $\mathcal{S}$ is defined as [1],

$$\mathcal{S}^* = \{\boldsymbol{\omega} | \boldsymbol{\omega}^T \boldsymbol{\sigma} \geq 0 \, \forall \boldsymbol{\sigma} \in \mathcal{S}\}$$

and for the second-order cone $\mathcal{K}$, it can be proved that $\mathcal{K}^* = \mathcal{K}$. The primal and dual problems are then obtained by keeping the maximisation or the minimisation at the left and right side of the second equality respectively. More explicitly, the primal problem in (1) may be deduced by taking derivatives of the Lagrangian with respect to the dual variables $(\mathbf{v}, \boldsymbol{\omega})$, while the dual form of the optimisation problem is obtained by taking derivatives of the Lagrangian with respect to the primal variables $(\mathbf{x}, \lambda)$, which results in,

$$\begin{aligned} \textbf{Dual: } \lambda^* = \min_{\mathbf{v}} \ & \mathbf{b}^T \mathbf{v} \\ s.t. \ & \mathbf{f}^T \mathbf{v} = 1 \\ & -\mathbf{X}^T \mathbf{v} \in \mathcal{K}^* \end{aligned} \tag{4}$$

3 R-adaptivity

3.1 Lower bound problem

We aim to further increase the value of λ^* by varying the nodal positions $\mathbf{x}$. This corresponds to adding a further maximisation in (3), which now reads,

$$\lambda^{LB} = \max_{\mathbf{x}} \max_{\boldsymbol{\sigma}, \lambda} \min_{\boldsymbol{\omega} \in \mathcal{K}^*, \mathbf{v}} \mathcal{L}(\boldsymbol{\sigma}, \lambda, \mathbf{x}; \mathbf{v}, \boldsymbol{\omega})$$

From this expression, the following primal problem is deduced,

$$\begin{aligned} \lambda^{LB} = \max_{\mathbf{x}, \lambda, \boldsymbol{\sigma}} \ & \lambda \\ s.t. \ & \mathbf{X}\boldsymbol{\sigma} + \lambda \mathbf{f} = \mathbf{b} \\ & \boldsymbol{\sigma} \in \mathcal{K} \end{aligned} \tag{5}$$

It can be observed that the equality constraints above have become non-linear on the variables $\boldsymbol{\sigma}$, λ and $\mathbf{x}$, and therefore the optimisation problem is highly non-linear, and not a SOCP anymore. However, given a set of primal-dual solution $(\lambda_k, \boldsymbol{\sigma}_k, \mathbf{x}_k; \mathbf{v}_k, \boldsymbol{\omega}_k)$, the Lagrangian may be linearised as follows:

$$\mathcal{L}(\boldsymbol{\sigma}, \lambda, \mathbf{x}; \mathbf{v}, \boldsymbol{\omega}) \approx \lambda + \mathbf{v}^T(\mathbf{b} - \mathbf{X}_k \boldsymbol{\sigma} - \lambda \mathbf{f}_k) - \boldsymbol{\omega}^T \boldsymbol{\sigma} - \mathbf{v}^T \left(\frac{\partial \mathbf{X}_k}{\partial \mathbf{x}} \boldsymbol{\sigma}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}} \lambda_k \right) \delta \mathbf{x} \tag{6}$$

with $\delta \mathbf{x} = \mathbf{x} - \mathbf{x}_k$, and $\mathbf{X}_k$ denotes matrix $\mathbf{X}$ evaluated for the nodal positions $\mathbf{x}_k$. The approximated Lagrangian gives rise to the following primal problem:

$$\boxed{\begin{aligned} \textbf{Primal(LB)-}\delta: \ \lambda^{LB} = \max_{\delta \mathbf{x}, \lambda, \boldsymbol{\sigma}} \ & \lambda \\ s.t. \ & \mathbf{X}_k \boldsymbol{\sigma} + \left(\frac{\partial \mathbf{X}_k}{\partial \mathbf{x}} \boldsymbol{\sigma}_k + \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}} \lambda_k \right) \delta \mathbf{x} + \lambda \mathbf{f} = \mathbf{b} \\ & \boldsymbol{\sigma} \in \mathcal{K}, ||\delta \mathbf{x}|| \leq \boldsymbol{\epsilon} \end{aligned}} \tag{7}$$

This problem has linear constraints. Furthermore, the constraint $||\delta \mathbf{x}|| \leq \boldsymbol{\epsilon}$ which is imposed in order to avoid elements with negative Jacobian, may be recasted as a second-order cone, which renders the problem in (7) as a SOCP. The matrices $\frac{\partial \mathbf{X}_k}{\partial \mathbf{x}}\boldsymbol{\sigma}_k$ and $\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\lambda_k$ may be approximated by using numerical differentiation as follows:

$$
\begin{aligned}
\frac{\partial \mathbf{X}_k}{\partial x_i}\boldsymbol{\sigma}_k &\approx \frac{(\mathbf{X}_{k+\delta x_i} - \mathbf{X}_k)\boldsymbol{\sigma}_k}{\delta x_i} \\
\frac{\partial \mathbf{f}_k}{\partial x_i}\lambda_k &\approx \frac{(\mathbf{f}_{k+\delta x_i} - \mathbf{f}_k)\lambda_k}{\delta x_i}
\end{aligned}
$$

where matrix $\mathbf{X}_{k+\delta x_i}$ and vector $\mathbf{f}_{k+\delta x_i}$ denote $\mathbf{X}$ and $\mathbf{f}$ evaluated with the nodal coordinate x_i^k perturbed by a small quantity δx_i.

Physical Interpretation The constraints in (7) are in fact equivalent to imposing the equilibrium constraints on a moving mesh, such that the stresses and the final position of the mesh are unknown. Due to the non-linearity of these constraints, these equilibrium equations are linearised at the previous stress values $\boldsymbol{\sigma}_k$ and previous nodal positions, which gives rise to the approximated equilibrium constraints in (7).

3.2 Upper bound problem

In contrast to the lower bound problem, we aim to minimise λ^* with respect to the nodal positions $\mathbf{x}$, that is,

$$
\lambda^{UB} = \max_{\boldsymbol{\sigma},\lambda} \min_{\mathbf{x}} \min_{\boldsymbol{\omega}\in\mathcal{K}^*,\mathbf{v}} \mathcal{L}(\boldsymbol{\sigma}, \lambda; \mathbf{v}, \boldsymbol{\omega}, \mathbf{x})
$$

The nodal positions should be gathered with the dual variables. Thus, given a set of primal-dual solution $(\lambda_k, \boldsymbol{\sigma}_k; \mathbf{v}_k, \boldsymbol{\omega}_k, \mathbf{x}_k)$, we approximate the Lagrangian as,

$$
\mathcal{L}(\boldsymbol{\sigma}, \lambda; \mathbf{v}, \boldsymbol{\omega}, \mathbf{x}) \approx \lambda + \mathbf{v}^T(\mathbf{b} - \mathbf{X}_k\boldsymbol{\sigma} - \lambda\mathbf{f}_k) - \boldsymbol{\omega}^T\boldsymbol{\sigma} - \mathbf{v}_k^T\left(\frac{\partial \mathbf{X}_k}{\partial \mathbf{x}}\boldsymbol{\sigma} + \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\lambda\right)\delta\mathbf{x} \tag{8}
$$

From this expression, the following dual problem may be derived,

$$
\begin{aligned}
\textbf{Dual(UB)-}\delta: \ \lambda^{UB} = &\min_{\mathbf{v},\delta\mathbf{x}} \mathbf{b}^T\mathbf{v} \\
&s.t.\ \mathbf{f}^T\mathbf{v} + \left(\mathbf{v}_k^T\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\right)\delta\mathbf{x} = 1 \\
&\quad - \mathbf{X}_k^T\mathbf{v} - \left(\frac{\partial \mathbf{X}_k^T}{\partial \mathbf{x}}\mathbf{v}_k\right)\delta\mathbf{x} \in \mathcal{K}^* \\
&||\delta\mathbf{x}|| \leq \boldsymbol{\epsilon}
\end{aligned} \tag{9}
$$

where again, we have added the last constraint in order to avoid elements with large aspects ratios or negative Jacobian. This is an extension of the dual in (4) for varying nodal positions $\mathbf{x}$. The primal form of (9) reads,

$$
\begin{aligned}
\textbf{Primal(UB)-}\delta: \ \lambda^{UB} = &\max_{\lambda,\boldsymbol{\sigma}} \lambda \\
&s.t.\ \mathbf{X}_k\boldsymbol{\sigma} + \lambda\mathbf{f} = \mathbf{b} \\
&\quad \left(\mathbf{v}_k^T\frac{\partial \mathbf{X}_k}{\partial \mathbf{x}} + \mathbf{v}_k^T\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}\right)\boldsymbol{\sigma} \in \mathcal{K}_x \\
&\boldsymbol{\sigma} \in \mathcal{K}
\end{aligned} \tag{10}
$$

where the second constraints follows from deriving $\delta\mathbf{x}$ with respect to the dual variable $\delta\mathbf{x}$, and $\mathcal{K}_x$ is a cone equivalent to the constraint $||\delta\mathbf{x}|| \leq \epsilon$.

3.3 Strategy

3.3.1 Combining upper and lower bound solution

The analytical solution of the limit analysis yields a unique value of λ^*, but not necessarily a unique mechanism. For this reason, and due to the finite element discretisation, the optimal nodal positions $\mathbf{x}$ will differ in the upper and lower formulations. We choose to modify the nodal positions according to the average of the two values of $\delta\mathbf{x}$ obtained in each case. More explicitly, if $\delta\mathbf{x}^{UB}$ and $\delta\mathbf{x}^{LB}$ are the optimal solutions of (7) and (10), the mesh is updated according to the following vector:

$$\delta\mathbf{x} = \frac{1}{2}\left(\delta\mathbf{x}^{UB} + \delta\mathbf{x}^{LB}\right)$$

or an wighted direction according to the gain in each bound,

$$\delta\mathbf{x} = \left(\Delta\lambda^{UB}\delta\mathbf{x}^{UB} + \Delta\lambda^{LB}\delta\mathbf{x}^{LB}\right)/\Delta\lambda$$

3.3.2 Future work: combining other remeshing strategies

The adaptive solution described here may be combined with other strategies such as embedded remeshing [8]. In this reference, the number of elements is always increases. Therefore, in order to use the linearised forms of the Lagrangian in (6) or (8) for the lower an upper bound, it will be necessary to project the stresses (lower bound) or the velocities (upper bound) onto the new mesh.

In addition, due to the localisation of the plastic zone, it may be advised to allow the nodes to move only along a reduced portion of the domain, thus reducing the cost of the complete optimisation problem.

4 Conclusions

The optimisation problem has been here expanded with the nodal positions as additional variables, which further improves the bounds. The additional cost may be reduced by just adding in the optimisation process the position of those nodes that contribute to the failure mechanism.

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Abstract proposal for 5th Workshop on Direct Methods 2015

Numerical yield design analysis of high-rise reinforced concrete walls in fire conditions

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Abstract

The present contribution aims at predicting the failure of high rise reinforced concrete walls subjected to fire loading conditions. The stability of such structures depends, on the one hand, on thermal strains inducing a curved deformed configuration and, on the other hand, on a local degradation of the constitutive material strength properties due to the presence of a temperature gradient across the wall thickness. A three step procedure is proposed, in which the yield design (limit analysis) method is applied on two separate levels. First, an up-scaling procedure on the wall unit cell is considered as a way for assessing the generalized strength properties of the curved wall, modeled as a shell, by taking into account reduced strength capacities of the constitutive materials. Secondly, the overall stability of the wall in its fire-induced deformed configuration is assessed using lower and upper bound shell finite elements and the previously determined temperature-dependent strength criterion. Second-order cone programming problems are then formulated and solved using state-of-the-art solvers. Different illustrative applications are presented to investigate the sensitivity of the wall stability to geometrical parameters. Finally, the influence of imperfect connections between pannels is also considered using a simple joint behavior.

Keywords : Yield design; Limit analysis; Fire conditions; Shells; Finite Elements

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Shakedown analysis of 3D frames for complex statical and seismic load combinations

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Keywords: shakedown, 3D frames, seismic analysis.

Structures, during their operational life, are subjected to a sequence of variable actions, including dead, anthropic and natural loads. Building codes fix the range and extension of load variability through combination formulas that usually involve a large number of load conditions. The loading scenario can become very complex in presence of seismic actions taken into account through nonlinear modal combination rules such as the well known SRSS or CQC rules.

In this context shakedown analysis furnishes, in a direct way, a reliable safety factor against plastic collapse, loss in functionality due to excessive deformation (ratcheting) or collapse due to low cycle fatigue (plastic shakedown), and also provides valuable information about the internal stress redistribution due to the plastic adaptation phenomenon [1].

Despite its technical implications, nowadays shakedown analysis still seems confined to the research community instead of being a common tool in structural design. This is largely due to the difficulties in managing the large number of load conditions and the complex combination rules required by widely employed building codes which greatly increase the solution costs and prevent analysis.

In the paper, with respect to 3D frames, a strategy for an efficient treatment of complex statical and seismic load combinations is proposed with the aim of making the shakedown analysis an affordable design tool to be used in practical applications. An algorithm capable of detecting a small number of significant elastic stresses within those corresponding to the load domain and suitable for use in the case of response spectrum analyses, is proposed.

The yield surface of the beam sections is defined by its support function values associated to presso-flexural mechanisms and it is approximated as Minkowski sum of ellipsoids. The analysis is performed on the basis of the algorithms proposed in [2,3]. A series of numerical tests are presented to show both the accuracy and the effectiveness of the strategy.

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Study of effective strengths of particulate reinforced metal matrix composites (PRMMCs) using direct method and statistical learning

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Abstract

Particulate reinforced metal matrix composites (PRMMCs) are characterized by their random and irregular microstructure. Morphological features of the material have been acknowledged to have great impact over their effect behaviors, above all the material strength. In order to exploit how morphological feature interact with material behaviors, we elaborated in present study a numerical approach which incorporates homogenization technique, shakedown analysis, and statistical learning. To demonstrate this approach, a typical PRMMC material, WC - 20 Wt.% Co, was taken as example. Numerous representative volume element (RVE) samples of this material were built based on both real and artificial microscope images. Multiple effective behaviors, including the ultimate strength and endurance limit, were evaluated on each RVE sample. By collecting the result data and applying statistical models such as regression and artificial neural network (ANN) on them, factors contributing to overall material parameters and strengths were identified and investigated in detail.

Keywords: particulate reinforced metal matrix composite (PRMMC), direct method, statistical learning, representative volume element (RVE), micromechanics

SHAKEDOWN ANALYSIS UNDER STOCHASTIC UNCERTAINTY BY CHANCE CONSTRAINED PROGRAMMING

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Key Words: *Limit Analysis, Shakedown Analysis, Reliability Analysis, Chance Constrained Programming, Non-linear Optimization, smoothed finite element method.*

The plastic collapse limit and the shakedown limit which define the load-carrying capacity of structures are important in assessing the structural integrity. Due to the high expenses of experimental setups and the time consuming full elastic-plastic cyclic loading analysis, the determination of these limits by means of numerically direct plasticity methods has been of great interest to many designers. Moreover, a certain evaluation of structural performance can be conducted only if the uncertainty of the actual load-carrying capacity of the structure is taken into consideration since all resistance and loading variables are random in nature. As the result of the need to account in a rational way for such uncertainties, the theory of structural reliability has been introduced and has developed rapidly. Limit and shakedown analysis render the stochastic problem time invariant [e.g. 1-3,5-6].

Chance constrained programing is an approach of stochastic programming to limit and shakedown analysis under uncertainty. Under uncertainty the shakedown problem can be stated with random objective function or with random constraints, a probability is set with which the constraint has to be satisfied [1-2]. The edge-based smoothed finite element method (ES-FEM) was recently proposed to significantly improve the accuracy and convergence rate of the standard finite element formulation for static, free and forced vibration analyses of solids. It also was applied successfully in shakedown analysis of structures [4-5].

In this study, we present a new primal-dual chance constraint algorithm of shakeown problems under uncertainty. In this investigation, we restrict ourself to the case of random yield limit, the loads applied to structures are still deterministic. Using chance constrained programming, we get deterministic equivalent formulations based on upper bound and lower bound theorems and then prove that the formulations are actually dual to each other. As aforementioned, in the study described here the numerical approach is based on the ES-FEM method. In the ES-FEM, compatible strains are smoothed over the smoothing domains associated with edges of elements. Using a constant smoothing function, only one Gaussian point is required for each domain ensuring that the total number of variables in the resulting optimization problem is kept to a minimum compared with standard finite element formulation. In this study, three-node linear triangular elements are used to analyze plane stress problems.

Some numerical examples are investigated to test the proposed algorithm. The obtained solutions match well with analytical values and show remarkably good performance.

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A direct solution technique and bounds for the steady state response of elastic-plastic structures on the basis of a discrete primal-dual formulation

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Keywords: direct steady cycle computation, finite elements, plastic ratcheting.

Whereas a structure usually shakes down to purely elastic deformation over a small number of cycles, convergence to a steady state in excess of shakedown can proceed quite slowly and take many tens of cycles. In such case the whole stress-strain history can be cumbersome to compute using the step-by-step approach. Moreover, it gives no reliable evidence of convergence to the steady state. Although effective direct procedures for elastic and plastic shakedown have already been created and implemented, there are only few approaches for steady plastic response in the literature, such as [1], which iteratively adjusts residual stresses. Alternatively, the problem of determining steady plastic response under prescribed cyclic loading can be formulated as a constrained optimization problem [2, 3].

The current study derives formulations for the steady cycle problem in the framework of a discrete form of a primal-dual approach. The formulation for a structure made of a von Mises material is cast in the form of a symmetric convex primal-dual problem. The dual gap provides a certificate of convergence, and the comparison of primal and dual cycles enables one to estimate the quality of time discretization. However, this primal-dual formula is singular in the meaning that, on the one hand, if stress achieves the yield surface, the plastic strain magnitude is undetermined from the flow rule. On the other hand, with the knowledge of the plastic strain, the flow rule does not determine the spherical part of the stress tensor.

The first singularity is treated by smoothing the sum of the norms in the object function similarly to regularization made in [4] for the elastic shakedown problem. Such regularization leads to a system of equations, which is solvable if volumetric plastic strain can be eliminated in the formulation as it happens in plane and uniaxial stresses. In view of a great number of unknowns in the steady cycle problem, an iterative solution method of the linearized equation system is proposed. The method proceeds by solving sequentially the linear systems, each having dimension of the number of degrees of freedom only. For testing, this technique was successfully applied for solving a pressurized thin-walled tube with thermal through-wall transients (the Bree problem). The computations have demonstrated a rapid and stable convergence to steady cyclic states.

In order to extend the technique to triaxial stress, a modified primal-dual formulation has been derived, in which material is allowed to yield volumetrically with resistance to such deformation being controlled by a penalization parameter in the formula. On increasing the parameter’s value, the material behaviour ultimately tends to be incompressible. The formulation was tested by solving the same problem without imposing the strict incompressibility condition, and similar results were obtained.

Thus, the formulation of the steady cycle problem for a discretized body as a symmetric primal-dual optimization problem with strong duality has led to a formula, which can be solved efficiently with an intuitively apprehensible iterative direct technique. The method has been extended to a general stress state by allowing small volumetric plastic straining. The difference between primal and dual steady cycle parameters (plastic strains and stresses) can be used for the assessments of time discretization quality, and a vanishing dual gap – as an evidence of solution convergence.

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Micromechanical modelling on plastic and creep behaviours of MMCs using the Linear Matching Method

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Keywords: Linear Matching Method (LMM); Creep; Fatigue; Cyclic Plasticity; Metal Matrix Composite (MMC).

The main purpose of this work is to present the results obtained by the Structural Design and Life Assessment Research Group on the plastic and creep structural response of Metal Matrix Composite (MMC) using the Linear Matching Method (LMM). Throughout the literature different approaches for modelling fibre or particles reinforced aluminium alloy MMCs have been published [1-9]. Nonlinear Finite Element Analyses were used to describe particular MMCs behaviours, such as deboning, cyclic damage accumulation and creep rupture at high temperature. As a powerful Direct Method the LMM has demonstrated its capability of assessing both the shakedown and ratchet limits [10-13], the creep rupture limit [14, 15], and creep fatigue assessment [16-19] in a very accurate, robust and efficient way.

The long fibre reinforced MMC unit cell is identified due to the periodic dispersion of fibres within the matrix. The MMC is subjected to a cyclic thermal load and to a constant mechanical one. When creep needs to be considered, a dwell period Δt is introduced within the load cycle. The numerical model consists of a quarter of the unit cell with two planes of symmetry applied and plain condition on the two external faces to introduce the periodic constraints. The first part of this work investigated the effects of different fibre materials (SiC and Al_2O_3) and fibre volume fraction on the MMC's plastic behaviour. It has been demonstrated [20] that the increase of the fibre volume fraction over the 40% is beneficial to the load bearing capacity of the MMCs. In opposition to this a small reduction of the reverse plasticity and ratchetting limit is observed. For the Al/SiC and Al/ Al_2O_3 the plastic and ratchetting strain ranges have been calculated for a large variety of cyclic temperature. In order to evaluate the maximum plastic strain range a unique material independent formulation has been introduced for low cycle fatigue assessment.

In the second part the results for the creep fatigue response are presented [21], considering the effects of dwell time, reverse plasticity and stress relaxation on the unit cell life. The MMC responses are evaluated for different cyclic load points within the shakedown limit. Three principal failure mechanisms are identified and discussed accurately, cyclic enhanced creep, thermal fatigue and creep ratchetting. For creep-fatigue interaction the two most important parameters are the primary load level and the dwell time. If the creep dwell is relatively small, 1 hour in this case, a closed hysteresis loop is observed for a moderate level of primary load. The increase of dwell time enhances the total strain range by allowing a larger plastic strain range during the unloading phase. When a steady state stress occurs at the end of the creep dwell this mechanism stops. If the dwell time is further increased the inelastic strains during the loading and creep phase cannot be compensated by the reverse plasticity and creep ratchetting occurs. For any increase of the dwell time the creep ratchetting phenomenon becomes dominant, this scenario is identified as the most damaging for large primary load. Contrary when the primary load is zero or negligible, creep ratchetting cannot occurs and creep dwell affect only the total strain range enhancing the thermal fatigue damage. The accuracy and the efficiency of the LMM for the evaluation of creep fatigue damage are confirmed by ABAQUS nonlinear step by step analyses. This comparison further demonstrates the LMM capacity of providing accurate solution reducing the computational costs.

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Shakedown state in polycrystals: a direct numerical assessment

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It is well known that in high cycle fatigue (HCF), macroscopically, structures undergo elastic shakedown and the stress level (shear amplitude and hydrostatic pressure) commonly determines the lifetime. In this domain, the fatigue phenomenon is due to local plasticity at the grain scale. Therefore, some multiscale HCF multiaxial fatigue criteria were proposed, among them the well-known Dang Van criterion [Dang Van 1999]. This criterion supposes that in a polycrystal, some misoriented grains can undergo cyclic plasticity (plastic shakedown), which conduct to crack initiation.

The objective of this work is to validate this assumption by conducting numerical simulations on polycrystalline aggregates. As it is necessary to estimate the stabilized state in each grain of the polycrystal, classical incremental simulations are not the best way as it will be highly time-consuming because of the size of the aggregate. In the recent years, Pommier proposed a method called Direct Cyclic Algorithm to obtain the stabilized response of a structure under cyclic periodic loading, which it is shown to be more efficient compared to an incremental analysis in such situation [Pommier 2003]. However, errors can be obtained in certain case with respect to the incremental solution.

In this work, Crystal Plasticity FEM models, based on dislocation densities and large deformation [Seghir 2012], were used. As a first step, an aggregate of 20 grains of AISI 316L stainless steel under strain controlled cyclic loading was studied. Precise comparisons were conducted with incremental analysis and the results show that DCA seems to be an efficient solution in order to estimate the shakedown state of polycrystalline aggregates.

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Effective strength of wood cells determined by means of numerical limit analysis

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Abstract

The excellent mechanical and physical properties of wood combined with the general trend of growing environmental awareness in civil engineering have led to an increasing demand for wooden building structures in recent years. Nevertheless, wood, as a structural bearing material, is often countered skepticism because, due to its quite complex material behavior, introduced by its strong anisotropy and growth-induced inhomogeneities (like knots), prediction tools for the load carrying capacity of wood products have so far not achieved the accuracy of those from other building materials.

This is the main motivation for the proposed work, which aims at a new approach to understand and estimate failure mechanisms and the strengths of wood. Since failure initiation and crack formation is strongly influenced by the complex material system of wood, exhibiting cellular and layered structures on different length scales, a mechanical concept, in which these different microstructural characteristics are incorporated, appears to be necessary. Thus, the division of wood into meaningful levels of observation was the first objective of the proposed work. At each level, failure modes and failure stress states (strength properties) are to be determined, and the obtained information is to be transferred - and will serve as input - to the next higher level of observation.

For this reason, within this work, numerical limit analysis concepts were extended to meet the requirements of wood behavior and applied on wooden microstructures for the first time. This numerical method, known as direct method, exclusively focuses on the time instant of failure, and thus allows a very efficient prediction of lower- and upper bounds for effective strengths. A key component of this method is a nonlinear optimization problem, which, thanks to rapidly revolutions of computer technology and the developments of mathematical programming, can be solved efficiently in the form of second order cone programming (SOCP) [3–5]. The nature of the SOCP allows a wide range of convex functions to be defined for the plastic failure behavior. The corresponding problems are solved by the commercial software MOSEK [2], which is based on one of the leading algorithm of SOCP, the interior-point method developed by Andersen et al. [1].

In a first step, this numerical limit analysis concept is applied to early- and latewood separately, for which the significantly repetitive honeycomb structure can be approximated by two wooden unit cells with periodic boundary condition. Each unit cell exhibits a composite structure consisting of an isotropic layer, the middle lamella, and an anisotropic layer, the cell wall, described by the von Mises failure criterion and the Tsai-Wu failure criterion, respectively. For the discretization, linear triangular elements are employed for the lower bound approach and quadratic triangular elements are used for the upper bound case. In Figure 1a one illustrative failure mechanism of the latewood unit cell under macroscopic uniaxial loading in ***R*** direction is shown, where the failure propagates through corners of the cell wall layer and the middle lamella layer in ***T*** direction. By applying different loading combinations on this wooden unit cells, estimates of effective failure surfaces for early- and latewood and the corresponding failure mechanism could be obtained. Lower- and upper bounds of effective

failure surfaces under plane strain conditions in the *RT* plane are shown in Figure 1b, in which, each effective failure stress state corresponds to one simulation, also delivering a distinct failure mechanism.

In a next step, these results were transferred to the next higher scale, the clear wood, built up of early- and latewood layers. A comparison of numerical results with results from biaxial tests on clear wood shows a very good agreement, qualitatively as well as quantitatively. Finally, new insights into failure mechanisms of wood were gained and a new numerical tool for predicting wooden strength very efficiently could be developed.

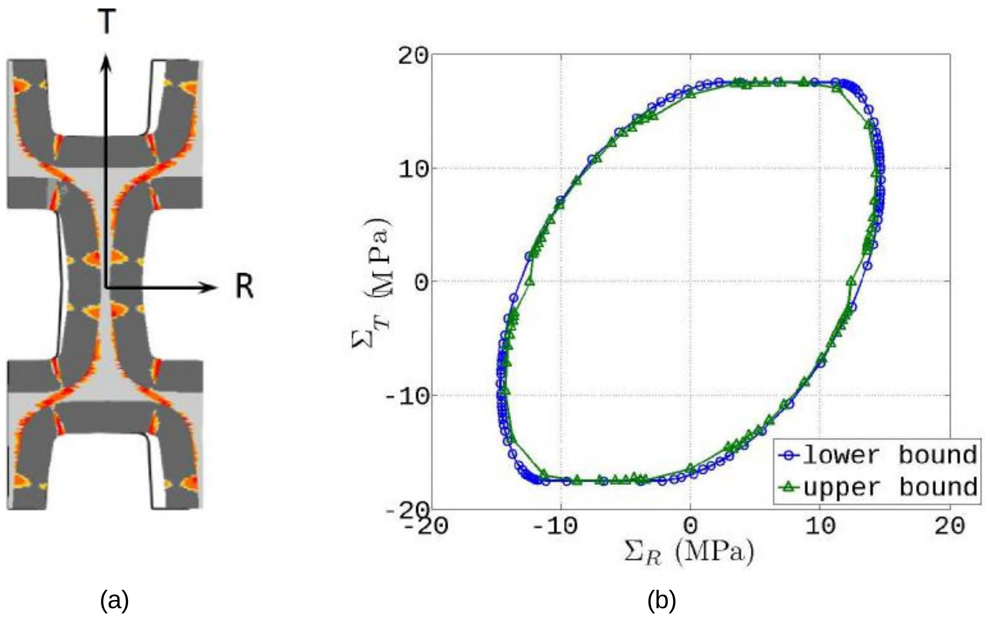

Figure 1: (a) Illustrative failure mechanism of a latewood unit cell obtained from upper-bound limit analysis approach, and (b) lower- and upper bound of the effective failure surface of latewood.

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Lower Bound Limit Analysis of defective pipelines under combined loadings

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Abstract. In this paper, a numerical procedure for plastic limit analysis of 3-D elastic-perfectly plastic and strain-strengthening bodies under complex loads is developed. The method is based on the lower-bound limit theorem and von Mises yield criterion so that the lower-bound limit analysis can be conducted by solving a nonlinear mathematical programming problem. A SQP algorithm and a dimension reduction-based technique are used to solve the discretized finite element optimization formulation. A conception of active constraint set is introduced, so that the number of constraints can be reduced greatly. The basis vectors of reduced residual stress spaces are constructed by performing an equilibrium iteration procedure of elasto-plastic finite element analysis. The numerical procedure is applied to carry out the plastic limit analysis of pipelines with part-through slots under internal pressure, bending moment and axial force. The typical plastic failure modes of defective pipelines are revealed. The effects of different sizes of part-through slots on the limit loads of pipelines are studied.