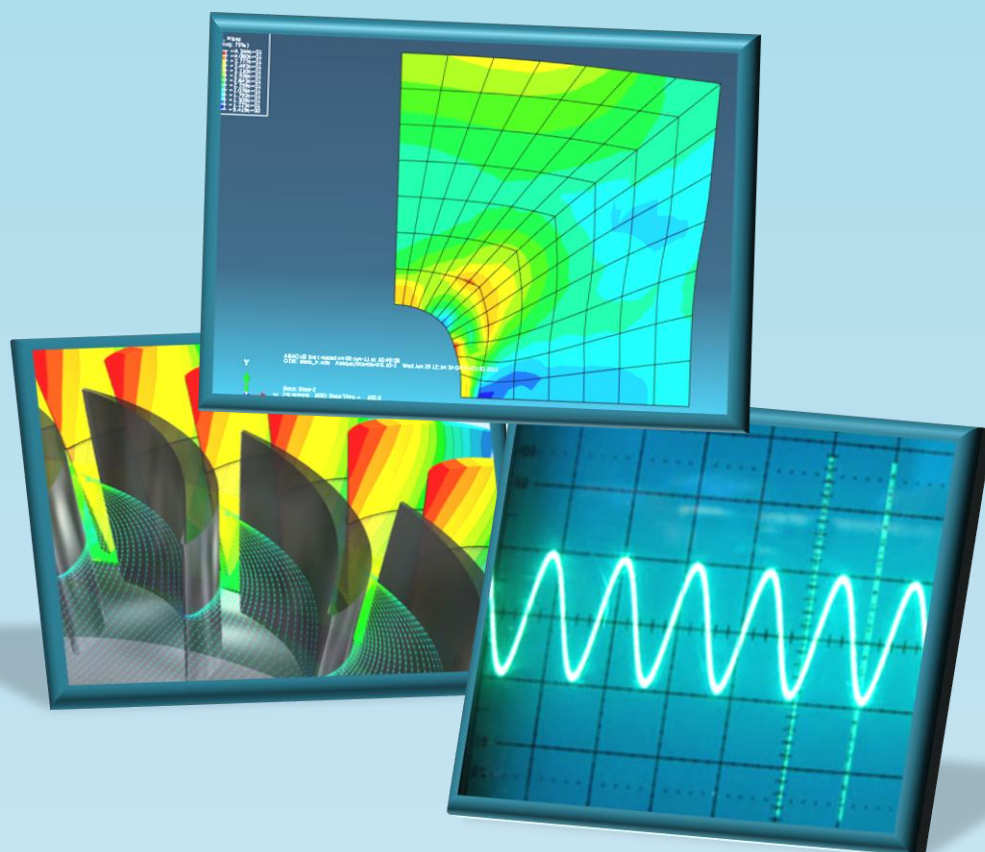




3rd International Workshop

on Direct Methods in Mechanics



A
B
S
T
R
A
C
T
S



National Technical University of Athens
School of Civil Engineering

Institute of Structural Analysis & Antiseismic Research
Zografou Campus, Athens, Greece

Third International Workshop on Direct Methods
NATIONAL TECHNICAL UNIVERSITY OF ATHENS

Final Schedule

Monday, 20th February 2012

- 8.30 Welcome at the Onassis Cultural Center, 107-109 Syngrou Avenue
- 8.50 Introduction by K.V. Spiliopoulos
- 9.00 F. Pastor, J. Pastor, D. Kondo
'Numerical limit analysis and hollow spheroid problems with Hill orthotropic matrix'
- 9.40 A. A. Pisano, P. Fuschi
'Limit analysis: a layered approach for composite laminates'
- 10.20 Coffee break
- 10.40 J.-W. Simon, D. Weichert
'Shakedown analysis of kinematically hardening structures in n-dimensional loading space'
- 11.20 J.J. Muñoz, S. Rabiei, A. Lyamin, A. Huerta
'Computation of bounds for anchor problems in limit analysis and decomposition techniques'
- 12.00 T. N. Tràn, M. Staat
'Shakedown analysis of Reissner-Mindlin plates using the edge-based smoothed finite element method'
- 12.40 M. Chen, A. Hachemi, D. Weichert
'Progress in plastic design of composites'
- 13.20 Lunch
- 15.00 K.V. Spiliopoulos, K.D. Panagiotou
'The Residual Stress Decomposition Method (RSDM): a novel direct method to predict cyclic elastoplastic states'
- 15.40 E. Charkaluk, R. Seghir, L. Bodelot, J.F. Witz, P. Dufrénoy
'Experimental tools for the analysis of shakedown and limit state on metallic polycrystals'
- 16.20 Coffee break
- 16.40 M. Gilbert, C. Smith, A. Tyas
'Computational limit analysis and design optimization: use of adaptivity to solve large-scale problems'
- 17.20 Break
- 20.30 Dinner at the Costis Palamas building of the Cultural Center/Lounge of the UOA.

Tuesday, 21st February 2012

- 8.30 Welcome at the Onassis Cultural Center, 107-109 Syngrou Avenue
- 8.40 G. de Saxcé, C. Long, D. Kondo
'Macroscopic modeling of porous non associated frictional materials'
- 9.20 J. Wang, H-S. Yu
'Three-dimensional shakedown solutions and their application to road pavement design'
- 10.00 G. Garcea, A. Bilotta, R. Casciaro
'Direct evaluation of the post-buckling behavior of slender structures through a numerical asymptotic formulation'
- 10.40 Coffee break
- 11.00 M.-A. A. Skordeli, C.D. Bisbos
'Shakedown analysis of steel structures under ellipsoidal variable load domain'
- 11.40 M. Gilbert, C. C. Smith, S. J. Hawksbee
'Discontinuity layout optimization (DLO)'
- 12.20 Z. Kammoun, H. Smaoui, J. Pastor
'A quasi-periodic approximation based model reduction for limit analysis of micropile groups'
- 13.00 Lunch
- 14.30 A.R.S. Ponter
'A ratchetting problem in materials science'
- 15.10 Coffee break
- 15.30 O. Barrera, A.C.F. Cocks, A.R.S. Ponter
'Evaluation of the peak load corresponding to pre-assigned design criteria in composite laminates by the Linear Matching Method'
- 16.10 Discussion: Future trends and co-operation
- 16.50 End of the workshop
- 17.00 Departure for the Acropolis Museum

“Numerical limit analysis and hollow spheroid problems with Hill orthotropic matrix”

Franck Pastor¹, Joseph Pastor² and Djimedo Kondo³

¹ Laboratoire LML, Université de Lille-1, 59 655 Villeneuve d'Ascq, France ² Laboratoire LOCIE, Université de Savoie, 73 376 Le Bourget du Lac, France ³ Institut D'Alembert, Université P. M. Curie, 75 005 Paris Cedex 05, France

e-mails: franck.pastor@skynet.be, joseph.pastor@univ-savoie.fr, djimedo.kondo@upmc.fr

Abstract

The well-known Gurson [1] isotropic criterion of ductile porous media has been obtained by studying a hollow sphere with a von Mises rigid plastic matrix subjected to uniform strain rate boundary conditions. Gurson's analysis consisted in the use of the kinematic approach of limit analysis (LA) to obtain an upper bound of the exact macroscopic criterion for isotropic porous materials. Later, following the kinematic limit analysis approach, several extensions of the Gurson criterion, mainly accounting for void-shape effects, have been proposed. Effects of plastic anisotropy of the matrix have been addressed first in [2] for spherical voids, and later in the case of spheroidal (prolate and oblate) voids by [3] and [4]. Very recently, first numerical limit analysis investigations on ductile materials with oblate cavities in an isotropic von Mises matrix have been done in [5]; in the resulting codes the mechanical problem has been cast into a conic programming problem solved with the very effective commercial code MOSEK [6].

The objective of the present talk is to provide numerical lower and upper bounds to the macroscopic criteria of porous media having Hill orthotropic matrix. The study focused first on the extension of the existing codes to the case of the anisotropic matrix. For the static limit analysis approach, the static codes has been easily modified by mean of a simple change of variables. This is not the case for the kinematic approach, mainly because of the velocity discontinuities allowed through any tetrahedron side. To overcome this difficulty, we have extended to the 3D homogenization problem the mixed formulation defined in [7] which had been applied to plane strain structures in von Mises and Gurson materials in [8]. Indeed, this mixed formulation requires only the plasticity criterion of the material, without any recourse to the expression of the unit dissipated power, whereas it is needed by the direct kinematic method used in [5] in the isotropic case.

For comparison purpose, both resulting codes are first applied to the Gurson problem for validation and comparison with previous results in the von Mises matrix case. Then, owing to the good performance of the new codes, we provide numerical upper and lower bounds of the macroscopic criterion in the case of hollow sphere and spheroid with a Hill orthotropic matrix. This allows to assess the recent corresponding theoretical criteria available in the above mentioned literature.

References

- [1] A. L. Gurson. Continuum theory of ductile rupture by void nucleation and growth - part I: yield criteria and flow rules for porous ductile media. *J. Eng. Mat. Technology*, 99:2-15, 1977.
- [2] A. A. Benzerga and J. Besson. Plastic potentials for anisotropic porous solids. *Eur. J. Mechanics/A Solids*, 20:397-434, 2001.
- [3] V. Monchiet, O. Cazacu, E. Charkaluk, and D. Kondo. Macroscopic yield criteria for plastic anisotropic materials containing spheroidal voids. *Int. J. Plasticity*, 24:1158-1189, 2008.
- [4] S.M. Keralavarma and A.A. Benzerga. A constitutive model for plastically anisotropic solids with non-spherical voids. *J. Mech. Phys. Solids*, 58:874-901, 2010.
- [5] F. Pastor, D. Kondo, and J. Pastor. Numerical limit analysis bounds for ductile porous media with oblate voids. *Mech. Res. Comm.*, 38:350-354, 2011.
- [6] MOSEK ApS. C/O Symbion Science Park, Fruebjergvej 3, Box 16, 2100 Copenhagen 0, Denmark. Email: info@mosek.com, 2011.
- [7] F. Pastor. *Resolution par des méthodes de point intérieur de problèmes de programmation convexe par l'analyse limite*. Thèse de doctorat, Facultés universitaires Notre-Dame de la Paix, Namur, 2007.
- [8] F. Pastor, E. Loute, J. Pastor, and M. Trillat. Mixed method and convex optimization for limit analysis of homogeneous Gurson materials: a kinematical approach. *Eur. J. Mechanics A/Solids*, 28:25-35, 2009.

“Limit analysis: a layered approach for composite laminates”

Aurora Angela Pisano, Paolo Fuschi

Dept. Patrimonio Architettonico e Urbanistico - PAU
University Mediterranea of Reggio Calabria, Italy
e-mail: aurora.pisano@unirc.it; paolo.fuschi@unirc.it

Abstract

Limit analysis plays an eminent role among the theoretical and numerical methods aimed at predicting the load bearing capacity of structures or structural elements. It gives rapid answers and becomes very effective and attractive for the design of many modern industrial prototypes often manufactured with materials whose constitutive behaviour does not have a well defined mathematical description. In composite laminates, for example, many constitutive models are affected by values of material parameters hardly identifiable via experimental tests so that, in this context, results obtained by a step-by-step post-elastic analysis may be useless for applications of engineering interest.

The classical approaches of limit analysis rely upon mathematical programming procedures which have progressed significantly in recent years. Nevertheless, grounding on classical theorems of plasticity, many of these procedures are effective and computationally competitive only within the realm of perfect plasticity. A number of contributions (see e.g.[1] and references therein for the more recent ones) adopt a different approach whose basic assumption is that limit state solutions may be developed from sequences of elastic (linear) analyses easy to handle via any commercial finite element code. The further assumption of a Tsai-Wu type yield surface and of a non-associate flow rule allows the extension of limit analysis in a non-standard formulation.

In this context a numerical FE-based approach has been recently proposed by the authors to a peculiar problem of orthotropic composite laminates, namely the evaluation of the load bearing capacity of a pinned-joint composite plate [2]. In particular, two well known methods have been rephrased: the *Linear Matching Method*, conceived by Ponter and co-workers [3], has been used to compute an *upper bound* to the collapse load multiplier; the *Elastic Compensation Method*, due to Mackenzie and Boyle [4], has been employed to evaluate a *lower bound* to the collapse load multiplier. The former, considering a structure made by a fictitious linear viscous material with elastic parameters spatially varying, allows to construct a collapse mechanism and eventually to evaluate an upper bound to the collapse load. The latter, grounding on a stress redistribution procedure pursued by a sequence of elastic analyses in which highly loaded regions are systematically weakened, produces an admissible stress field suitable for a lower bound evaluation.

The present contribution summarizes some recent results obtained by the authors analyzing mechanically fastened joints in multi layers composite plates. The main novelty of such recent studies is of computational nature and it is not trivial being the *reformulation* of the above quoted limit analysis FE procedure *at layer laminate level*. One of the main simplifying assumptions of the previous works, namely that of an equivalent single layer laminate, is so released. Such a choice, as witnessed by the obtained results, allows to take into account the stacking sequence of the laminate and some of the through-thickness effects depending on the number of the laminate is made with and/or on the fiber orientations within each lamina, both affecting the joint bearing capacity. Higher order shell-type multilayered finite elements have been used, performing all the relevant operations at the Gauss points of each element layer (lamina). The layered approach reveals undoubtedly more burdensome computations but the comparison with a great number of experimental data, after [5,6], shows a good effectiveness of the proposed approach to tackle the addressed problem either in terms of estimation of the collapse load value or in terms of collapse mode and collapse zone prediction. The proposed procedure seems to guarantee, at least for the studied problem, a great accuracy for different laminate lay-ups and different joint geometries establishing an effective design tool useful to avoid expensive trials on real prototypes. The applicability to other mechanical problems, concerning the evaluation of load bearing capacity of composite laminates, seems to be straightforward.

References

- [1] Weichert, D., Ponter, ARS., Eds. *Limit States of Materials and Structures - Direct Methods*, Springer (2009).
- [2] Pisano, AA., Fuschi, P., Mechanically fastened joints in composite laminates: Evaluation of load bearing capacity, *Composites Part B: Engng*, **42**(4): 949-961 (2011).
- [3] Ponter, ARS., Carter, KF., Limit state solutions, based upon linear elastic solutions with spatially varying elastic modulus, *Comput Methods Appl Mech Eng*, **140**: 237-258 (1997).

- [4] Mackenzie, D., Boyle, JT., A method of estimating limit loads by iterative elastic analysis Parts I,II,III, *Int J Press Vessels and Piping*, **53**:77-142 (1993).
- [5] Okutan Baba, B., Behavior of pin-loaded laminated composites, *Experimental Mechanics*, **46**: 589-600 (2006).
- [6] Aktas, A., Dirikolu, MH., An experimental and numerical investigation of strength characteristics of carbon-epoxy pinned-joint plates, *Comp Sci and Tech*, **64**: 1605-1611 (2004).

“Shakedown analysis of kinematically hardening structures in n -dimensional loading spaces”

J.-W. Simon*, D. Weichert

Institute of General Mechanics, RWTH Aachen University,
Templergraben 64, D-52056 Aachen, Germany

*Current address: Institute of Applied Mechanics, RWTH Aachen University,
Mies-van-der-Rohe Str. 1, D-52074 Aachen, Germany,
jaan.simon@rwth-aachen.de

Keywords: Shakedown Analysis, Direct Methods, Interior-Point Algorithm, Limited Kinematical Hardening, Nonlinear Programming

Abstract

We consider engineering structures subjected to varying thermo-mechanical loading beyond the elastic limit. For these, we determine the shakedown factor a_{SD} , which is the maximum loading factor a such that the structure does neither fail due to spontaneous or incremental collapse nor due to alternating plasticity. This is done by means of direct methods, comprising limit and shakedown analysis. In particular, we follow the statical approach of MELAN [1], who formulated a shakedown theorem for elastic-perfectly plastic and unlimited kinematical hardening continua.

Consideration of kinematical hardening is important for many engineering problems and thus has been addressed by several authors in the field of shakedown analysis. Notably, accounting for only unlimited kinematical hardening does not cover incremental collapse but solely alternating plasticity. Here, we use the two-surface model proposed by WEICHERT and GROSS-WEEGE [2,3]. The kinematical hardening is considered as a translation of the yield surface in stress space, which is described by the six-dimensional vector of back-stresses π representing the motion of the yield surface's center, Fig. 1. Through the introduction of a second surface corresponding to the ultimate stress σ^H , this motion is bounded.

Using the statical shakedown theorem leads to nonlinear convex optimization problems, which are typically characterized by large numbers of unknowns and constraints. In this work, these will be solved by the interior-point algorithm IPSA recently developed by the authors [4]. This algorithm is capable of taking into account n -dimensional loading spaces, which until now has been only shown for perfectly plastic materials. In this paper, we will present the application of IPSA to structures with limited kinematical hardening subjected to more than two loads.

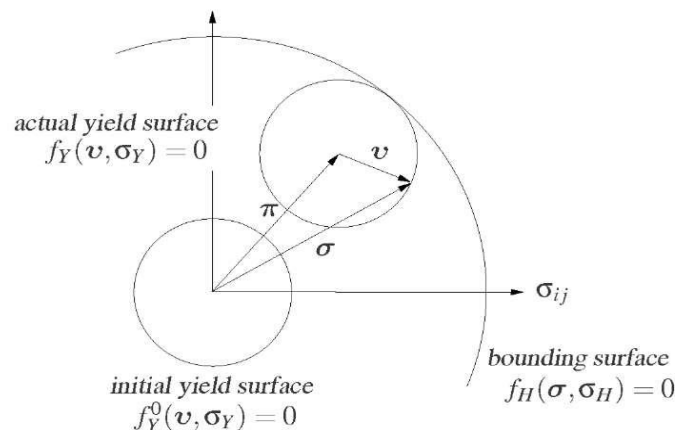


Figure 1: Kinematic hardening considered as translation of the yield surface in stress space

References

- [1] E. Melan. Zur Plastizität des räumlichen Kontinuums. *Ing-Arch*, 9:116-126, 1938.
- [2] D. Weichert and J. GroB-Weege. The numerical assessment of elastic-plastic sheets under variable mechanical and thermal loads using a simplified two-surface yield condition. *Int J Mech Sci*, 30(10):757-767,1988.
- [3] J. GroB-Weege and D. Weichert. Elastic-plastic shells under variable mechanical and thermal loads. *Int J Mech Sci*, 34:863-880,1992.
- [4] J.-W. Simon and D. Weichert. Numerical lower bound shakedown analysis of engineering structures. *Comput Methods Appl Mech Engrg*, 200:2828-2839,2011.

“Computation of bounds for anchor problems in limit analysis and decomposition techniques”

Jose J Muñoz¹, Seyednima Rabiei¹, Andrei Lyamin² and Antonio Huerta¹

¹LaCàN Research group, Dep. Applied Mathematics III, Universitat Politècnica de Catalunya (UPC)

²Center for Geotechnical & Materials Modelling, University of Newcastle, Australia

e-mail: j.munoz@upc.edu, nima.rabiei@upc.edu, Andrei.Lyamin@newcastle.edu.au, antonio.huerta@upc.edu

Abstract

1. LOWER AND UPPER BOUND FORMULATIONS

According to the lower (primal) and upper (dual) bound theorem of limit analysis, the bearing capacity of a structure is equal to (i) the maximum load factor λ^* under equilibrium conditions and with admissible stresses σ , (i.e. they belong to a set B), or alternatively (ii) equal to the minimum dissipation energy $D(v)$ as a kinematically admissible velocity field v .

By using specific interpolation spaces (σ^{UB}, v^{UB}) and (σ^{LB}, v^{LB}) we are able to compute exact upper and lower bounds of the optimal factor λ^* , denoted respectively by λ^{UB} and λ^{LB} (see [1] for specific details on the spaces). After applying such interpolations, the primal and dual problem turn into the following form,

$$\begin{aligned} \lambda^* &= \min_v D(v^{UB}) \\ \lambda^* &= \max \lambda \\ s.t. \quad &\begin{cases} A\sigma^{LB} + \lambda f = 0 \\ \sigma_i^{LB,e} \in B, \quad e = 1, \dots, N_e, i = 1, \dots, n_{sd} + 1 \end{cases} \end{aligned} \quad \begin{aligned} &s.t. \quad \begin{cases} \ell(v^{UB}) = 1 \\ -\varepsilon(v_i^{UB,e}) \in B^*, \quad e = 1, \dots, N_e, i = 1, 2 \\ -[v^{UB}]_i^\xi \otimes n^\xi \in B^*, \quad \xi = 1, \dots, N_\xi, i = 1, 2 \end{cases} \end{aligned}$$

The linear form $l(v)$, represents the dissipation energy due to the applied load. These problems can be solved efficiently using available optimisation programs. Moreover, usual plasticity criteria such as von Mises or Mohr-Coulomb in two dimensions allow us to rewrite the membership conditions as second order cones (SOC), which can be handled by the mentioned optimisation software. The optimum values of the lower and upper bound problem can be used to compute a set of elemental and edge contributions to the total gap, which are defined by:

$$\begin{aligned} \Delta\lambda^e &= \int_{\Omega^e} \sigma^{UB,e} : \varepsilon(v^{UB}) d\Omega + \int_{\Omega^e} \nabla \cdot \sigma^{LB} \cdot v^{UB} d\Omega - \int_{\partial\Omega^e} \sigma^{LB} n \cdot v^{UB} d\Gamma \\ \Delta\lambda^\xi &= \int_{\Gamma^\xi} s^{UB,\xi} \cdot [v^{UB}] d\Gamma - \int_{\Gamma^\xi} \sigma^{LB} n \cdot [v^{UB}] d\Gamma \end{aligned}$$

These bound gaps satisfy the properties, $\lambda^{UB} - \lambda^{LB} = \Delta\lambda^e + \Delta\lambda^\xi$, $\Delta\lambda^e \geq 0$, $\Delta\lambda^\xi \geq 0$, which make them good candidates to estimate the errors of the lower and upper bound solution. We have used them to design an adaptive remeshing strategy.

2. EXTENSION TO INTERFACES, DUPLICATED EDGES AND JOINTS

We will develop next specific conditions for common interface conditions encountered in geomechanics. In all cases we add specific constraints that preserve the strictness of the bounds. The studied and implemented situations are:

1. **Interface material** that splits two different materials with specific admissibility criterion for the common boundary.
2. **Duplicated edges**: in two-dimensional applications, it may convenient to overlap materials or structural elements such as ties or anchors. In these situations, it is required to have edges that joint one element on one side and two elements, B and B', on the other side.
3. **Modelling of joints** such as articulated joints in anchors and anchor-wall interface.

3. DOMAIN DECOMPOSITION

In order to reduce the memory requirement of realistic three dimensional problems, we propose a decomposition of the optimisation problems which consists in splitting our domain in two subproblems with local variables, and iteratively updating with a descent method a master variable t that in the LB problem corresponds to the internal tractions between the subdomains. Such variable may be seen as a (non-proportional) fictitious Neumann condition.

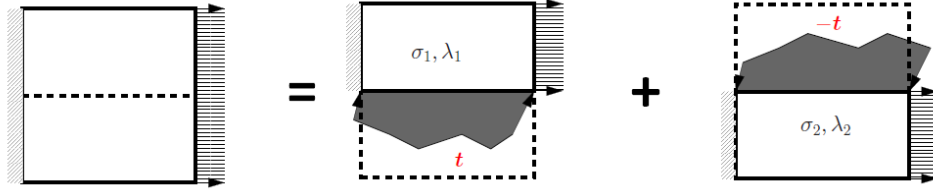


Figure 1. Illustration of the domain decomposition.

4. RESULTS

The extensions described in Section 2 have been employed to test the pull out capacity of multi-belled anchors, and determine the maximum height of simply supported and anchored retaining walls (see Figure 2). The linearity of the limit tension with respect to the number of bells has been verified. Five different anchor/soil conditions have been employed: rough (same properties as the soil), smooth (no resistance to shear), no tension condition, rough condition with no tension, and smooth condition with no-tension. Although the mechanisms do not significantly depend on these conditions, the pull out capacity does, and has been shown to be much larger for rough conditions. On the other hand, while for clay materials (zero internal friction angle, but non-zero cohesion) the failure mechanism is localised around the anchor (see Figure 2a), in other sand materials the slide-lines propagate up to the soil surface.

The computed limiting height agrees satisfactorily with experimental results and other numerical models that use incremental plasticity [2].

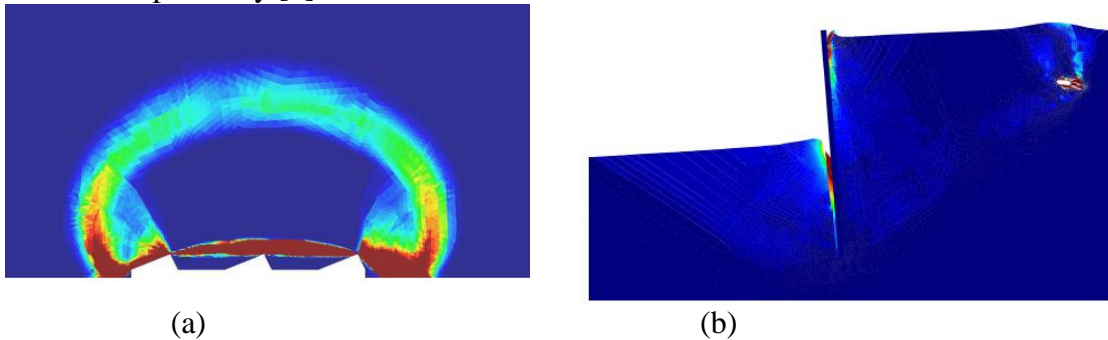


Figure 2. Contour plot of dissipation energy in (a) a three-bell anchor, and (b) of anchored retaining wall.

5. REFERENCES

- [1] H Ciria, J Peraire, and J Bonet. Mesh adaptive computation of upper and lower bounds in limit analysis. Int. J.Num. Meth. Engng., 75:899–944, 2008.
- [2] J J Muñoz, A V Lyamin, A Huerta. Stability of anchored sheet wall in cohesive-frictional soils by FE limit analysis. Submitted.

“Shakedown analysis of Reissner-Mindlin plates using the edge-based smoothed finite element method”

Thanh Ngọc Trần and M. Staat

Aachen University of Applied Sciences, Campus Jülich, Biomechanics Lab., Heinrich-Mußmann-Str. 1,
52428 Jülich, Germany, <http://www.fh-aachen.de/biomechanik.html>

Abstract

An edge-based smoothed finite element method (ES-FEM) for static, free vibration and buckling analyses of Reissner- Mindlin plates using triangular elements has been studied in [1]. In that method, the calculation of the system stiffness matrix was performed by supposing a constant strain field on the smoothing domains associated with edges of elements. The method was incorporated with the discrete shear gap (DSG) method together with a stabilization technique in order to avoid the transverse shear locking and to improve the accuracy of the numerical solution.

A primal-dual algorithm for shakedown analysis of Kirchhoff plates made of von Mises material has been recently developed using DKQ plate elements [2]. In this work, the algorithm will be further developed for calculating the plastic collapse limit and the shakedown limit of Reissner-Mindlin plates. The shakedown load multiplier formulated by static theorem will be proven actually to be the dual form of the shakedown load multiplier formulated by kinematic theorem by two propositions. According to the excellent performance in shakedown analysis of 2D structures by using ES-FEM [3], the method will be implemented with the technique developed in [1]. The method not only possesses all inherent features of convergence and accuracy from ES-FEM, but also ensures that the total number of variables in the optimization problem is kept to a minimum compared with the standard finite element formulation. Intensive numerical implementations show that the present method exhibits excellent convergence and accuracy of solutions from very thin plates to very thick ones.

References

- [1] Nguyen-Xuan H et al. (2010). An edge-based smoothed finite element method (ES-FEM) with stabilized discrete shear gap technique for analysis of Reissner-Mindlin plates. *Comput. Methods Appl. Mech. Engrg.* **199**(9-12): 471-489. <http://dx.doi.org/10.1016/j.cma.2009.09.001>
- [2] Thanh Ngoc Tran (2011). A dual algorithm for shakedown analysis of plate bending. *International Journal for Numerical Methods in Engineering*, **86**(7): 862–875. <http://dx.doi.org/10.1002/nme.3081>
- [3] Thanh Ngoc Tran, Liu GR, Nguyen-Xuan H, Nguyen-Thoi T (2010). An edge-based smoothed finite element method for primal-dual shakedown analysis of structures. *International Journal for Numerical Methods in Engineering* **82**(7): 917-938. <http://dx.doi.org/10.1002/nme.2804>

“Progress in plastic design of composites”

Min Chen, A. Hachemi, D. Weichert

Institut für Allgemeine Mechanik, RWTH Aachen

Abstract

To predict the failure of composites under monotonously increasing or variable loads with unknown loading history, direct methods, namely limit and shakedown analysis, are applied effectively on two levels [1, 2]: First, a representative volume element (RVE) is taken into account for the admissible safe local stress or strain domains. Second, based on the homogenization theory [3], the homogenized admissible macroscopic loading domains are evaluated. The influence of geometry and elastic-plastic material properties is focused for the investigation of properly design, which makes them increasingly attractive for demanding technical applications, like aerospace, automotive and marine industries.

The numerical tools are the finite-element method and non-linear optimisation. In our former work, non-conforming three-dimensional finite elements are used for the limit and shakedown analysis of periodic metal-matrix composites with the assumption that the behavior of the ductile phase is elastic-perfectly plastic. However, with the restrictions of optimization algorithms, shakedown theory has been developed for elastic-plastic hardening solids most theoretically. In this work, the static shakedown analysis with the consideration of limited kinematic hardening is implemented by using large scale interior-point-algorithm based optimization tool (IPOPT).

Furthermore, global homogenized material parameters are determined through fitting the admissible macroscopic limit stress domain under more complicate yield criteria, instead of von-Mises yield criterion.

References

- [1] Weichert, D.; Hachemi, A.; Schwabe, F.: Application of shakedown analysis to the plastic design of composites. *Arch. Appl. Mech.* 69: 9-10, pp.623-633, 1999.
- [2] Weichert, D.; Ponter, A.: *Limit states of Materials and Structures*. Springer, Wien/New-York, 2009.
- [3] Suquet, P.: *Plasticité et homogénéisation*. PhD. Thesis, Université Pierre et Marie Curie, Paris VI, 1982.

“The Residual Stress Decomposition Method (RSDM): A Novel Direct Method to Predict Cyclic Elastoplastic States”

K.V. Spiliopoulos, K.D. Panagiotou

*Institute of Structural Analysis and Antiseismic Research
National Technical University of Athens (NTUA)
Zografou Campus, 157 80 Athens, Greece
e-mail: kvspilio@central.ntua.gr, dinoscivil@hotmail.com*

Abstract

Civil engineering structures, like bridges, pavements, buildings, offshore structures are often subjected to high levels of cyclic loading due to heavy traffic, earthquake loading or waves. Mechanical engineering structures like nuclear reactors, aircraft gas turbine propulsion engines also operate in high levels of loads and temperature. Under all these kinds of loading all these structures are forced to develop plastic strains.

It is known that direct methods offer an alternative approach to time-stepping methods which need expensive calculations to compute the complete loading history of the above structures, especially with large number of degrees of freedom. These methods aim to approximate the possible stabilized states under repeated thermo-mechanical loading right from the start of the calculations.

A novel direct method is proposed in here which has its roots in a method that was proposed [1, 2] in the context of the cyclic loading analysis of creeping structures.

The material is assumed to behave as elastic-perfectly plastic. The irreversibility of the nonlinear material dictates the existence of residual stresses together with the elastic stresses. It is the distribution of the residual stresses that is sought at the cyclic stress state.

The proposed method is based on decomposing the unknown residual stresses in Fourier series of cosine and sine terms multiplied by unknown coefficients which are the ones we want to find. It is proved that these coefficients may be determined with the aid of the integral of the cycle time derivatives of these residual stresses inside a loading cycle. This derivative may be estimated at discrete cycle points by enforcing equilibrium and compatibility at these points. Should the total stress vector (the sum of the elastic and the residual stress vector) exceed at any of these points the yield surface, the plastic strain rate vector is estimated by returning, on the yield surface, not along the direction of the projection but along direction of the total stress vector. This ‘radial return’ type of mapping is quite straightforward to calculate for a von Mises yield surface, which is the yield surface considered herein. For other types of yield laws an analogous approach could be applied.

After we have discretized our structure with finite elements, an iterative numerical procedure is set up, whose steps may be briefly described as follows:

We start with calculating the elastic stress and elastic stress rates at some cycle point and we make an initial estimate of the Fourier coefficients

Using these coefficients we get an estimate of the residual stresses at a Gauss point (GP)

We find the total stress vector by adding the elastic and the residual stresses at this GP.

We check whether this stress vector has exceeded the yield surface. If this is the case we get an estimate of the plastic strain rate which will give us an unbalanced stress that is equilibrated by equivalent nodal forces.

Equivalent nodal forces are used to equilibrate the unbalanced stress. When added to the external loads a direct stiffness equilibrium rate problem is set up from which a displacement rate may be evaluated, using the standard stiffness matrix.

This displacement rate may be used together with the elastic stress rates and the plastic strain rates to get an estimate of the residual stress rates.

These rates are numerically integrated with time over the cycle points to obtain an update of the coefficients of the Fourier coefficients.

With the coefficients known one may get a new estimate of the residual stresses. If this estimate is close to the previous estimate, within a specified tolerance, we exit the iterative procedure; otherwise we go back to step c.

Once a steady state solution has been achieved, one checks whether, in this solution, any plastic strain rates exist at the GPs of the structure. If not, there is a condition of elastic shakedown at these GPs. If this condition holds for all the GPs of the structure, then the external loads have adapted the structure to an elastic shakedown state also.

If, on the other hand, plastic strain rates do exist, we check whether the integral over the cycle is different to zero or not. In case of a zero integral, the GPs are in a state of alternating plasticity.

If, on the other hand, this integral is different to zero, then we have conditions of ratchetting at these GPs. In case that this condition holds for so many GPs so that a mechanism is formed, the whole structure is in the case of incremental collapse.

Thus, following the proposed numerical approach, which was first presented just recently [3], one may determine what sort of cyclic elastoplastic state one would expect for a given cyclic loading, without having to follow a cumbersome time-stepping procedure.

The whole procedure appears to be computationally efficient. The stiffness matrix needs to be decomposed only once. Examples of application of one and two dimensional structures will be presented during the workshop.

References

- [1] K.V. Spiliopoulos, 'Simplified methods for the steady state inelastic analysis of cyclically loaded structures', D. Weichert, G. Maier (eds)., *Inelastic analysis of structures under variable loads*, Kluwer Academic Publishers, 213-232, (2000).
- [2] K.V. Spiliopoulos, 'A Simplified Method to Predict the Steady Cyclic Stress State of Creeping Structures', *ASME J. Appl. Mech.*, **69**,148-153, (2002).
- [3] K.V. Spiliopoulos, K.D. Panagiotou, 'A computational procedure for the cyclic steady state elastoplastic analysis of structures', *Proceedings COMPLAS XI*, 7-9 Barcelona, Spain, (2011).

“Experimental tools for the analysis of shakedown and limit state on metallic polycrystals”

E. Charkaluk^{*,†}, R. Seghir[†], L. Bodelot[†], J.F. Witz⁺ and P. Dufrénoy⁺

⁺ Laboratoire de Mécanique de Lille

Univ Lille Nord de France

boulevard Paul Langevin, 59655 Villeneuve d'Ascq, France

e-mail: eric.charkaluk@ec-lille.fr, web page: <http://www.univ-lille1.fr/lml>

[†] California Institute of Technology - GALCIT

1200 East California Boulevard, Pasadena, CA, 91125, US

Email: lbodelot@caltech.edu

Abstract

Polycrystalline metallic materials are made of an aggregate of grains more or less well oriented, with respect to the loading axis, for plastic gliding. Under mechanical loading, this leads to a heterogeneous deformation at the microstructure scale. This local plasticity triggers a heterogeneous thermal dissipation caused by mechanical irreversibility. Some original experimental works enabling the simultaneous determination of thermal and strain fields, in the same area, at this scale have already been realized in house on a A316L stainless steel [1, 2]. Two complementary ways are actually followed: some numerical treatments in order to access to experimental dissipations and the development of a consistent constitutive model. Both aspects are presented in this communication and a dialogue between microstructural texture coming from EBSD analysis, local deformation mechanism and thermal localisation phenomenon is introduced.

The treatment of the full-field measurements is done with respects to the polycrystalline texture of the material. Under different assumptions, strain and thermal fields are obtained grain to grain. The local disorientation, the different grain sizes and the crystallographic orientations are the main aspects, which seems to have an influence on the strain localization process at the grain scale. The analysis of the local temperature evolutions is a key important feature of this coupled analysis.

The numerical implementation in a FE code of a fully coupled crystalline plasticity constitutive model has been realised and is the other main part of this work [3, 4]. It enables to compare the local kinematic and thermal distributions during monotonic tests and to study the heterogeneity of the stored energy at grain scale. These analyses of thermomechanical couplings at the grain scale could lead to the definition of new thermodynamically based strain localization criteria.

References

- [1] L. Bodelot, L. Sabatier, E. Charkaluk, and P. Dufrénoy. Experimental set up for fully coupled kinematic and thermal measurements at the microstructure scale of an AISI 316L steel. *Mat. Sci. Eng. A*, 501:52–60, 2009.
- [2] L. Bodelot, E. Charkaluk, L. Sabatier and P. Dufrénoy. Experimental study of heterogeneities in strain and temperature fields at the microstructural level of polycrystalline metals through fully-coupled full-field measurements by Digital Image Correlation and Infrared Thermography. *Mech. Mat.*, 43(11):654-670, 2011.
- [3] R. Seghir, E. Charkaluk, P. Dufrénoy, L. Bodelot, Thermomechanical couplings in crystalline plasticity under fatigue loading, *Procedia Engineering*, 2(1):1155-1164
- [4] R. Seghir, L. Bodelot, E. Charkaluk, P. Dufrénoy, Numerical and experimental estimation of thermomechanical fields heterogeneity at the grain scale of 316L stainless steel, *Comp. Mat. Sci.*, accepted

“Computational limit analysis and design optimization: use of adaptivity to solve large-scale problems”

Matthew Gilbert¹, Colin Smith¹ and Andy Tyas¹

¹ *Computational Mechanics and Design Research Group, Department of Civil & Structural Engineering,
University of Sheffield, Mappin Street, Sheffield S1 3JD, UK*
m.gilbert@sheffield.ac.uk; c.c.smith@sheffield.ac.uk; a.tyas@sheffield.ac.uk

Abstract

To verify the safety of solid bodies and structures against collapse, engineers have traditionally had to rely either on simplistic hand type calculations, or on significantly more complex computational tools which identify the collapse state in an indirect, iterative manner. Such computational tools can be unreliable and often require large amounts of computer time and/or high levels of operator expertise. Additionally, in many engineering disciplines the initial design stage is carried out in an ad-hoc manner, with 'engineering intuition' often used to identify structurally efficient designs. Direct analysis and design methods can potentially address both these issues, and the mathematical similarities between analysis and design formulations means that a general integrated limit analysis and design optimization framework can be developed relatively easily, potentially allowing a broad range of problems of both industrial and academic interest to be solved.

Whilst developing an integrated limit analysis and design optimization framework recently at the University of Sheffield the decision was taken to make widespread use of highly efficient interior point linear programming solvers, in conjunction with an adaptive scheme to limit the size of problems being solved. The adaptive scheme entails initially solving a reduced problem and then successively adding variables or constraints to improve the solution (often termed 'column generation' or 'cut generation'). The adaptive scheme has been applied to various masonry and rock mechanics limit analysis problems involving combined rocking and crushing failure at joints (which gives rise to a non-linear failure envelope).

The adaptive scheme has also been applied with particular success to truss layout optimization (design) problems [1], with problems containing in excess of 10^9 potential truss bars being solved recently [2]. Since the solutions obtained are often extremely accurate, this can inspire the development of new analytical solutions to centuries old problems [3]. Additionally, the integrated nature of the general framework means that efficient strengthening schemes for under-strength structures can potentially be automatically identified ('retrofit design synthesis').

Finally, the study of both analysis and design problems permits similarities in the underlying mathematical formulations to be identified. This led the authors to realize that the truss layout optimization design technique could be modified so as to provide a powerful new direct analysis method for plasticity problems [4]; the technique, 'discontinuity layout optimization', will be described in more detail in another paper presented at this meeting.

References

- [1] M. Gilbert, A. Tyas. Layout optimization of large-scale pin-jointed frames. *Engineering Computations*; **20**; pp 1044-1064; 2003.
- [2] A. Pichugin, A. Tyas, M. Gilbert. On the optimality of Hemp's arch with vertical hangers. *Structural and Multidisciplinary Optimization*; accepted for publication, 2012.

- [3] A. Tyas, A.V. Pichugin, M. Gilbert. Optimum structure to carry a uniform load between pinned supports: exact analytical solution. *Proceedings of the Royal Society A*; **467**; pp 11011120; 2011.
- [4] C.C. Smith, M. Gilbert. Application of discontinuity layout optimization to plane plasticity problems. *Proceedings of the Royal Society A*; **463**; pp 2461-2484; 2007.

“Macroscopic modeling of porous non associated frictional materials”

Géry de Saxcé, Cheng Long

Laboratoire de Mécanique de Lille, UMR CNRS 8107, Université de Lille 1, Cité scientifique, F59655
Villeneuve d'Ascq, France

Djimedo Kondo

Institut Jean Le Rond d'Alembert, UMR CNRS 7190, Université Pierre et Marie Curie, 4 place Jussieu, F75005
Paris, France

Abstract

The present work can be considered as a first step to propose a macroscopic plastic model for “Porous non associated Drucker-Prager”-type materials, using homogenization techniques and the hollow sphere model proposed by Gurson [1] for von Mises solid matrix.

In the first part, we determine analytically the plastic limit state of a hollow sphere with a Drucker-Prager matrix and subjected to hydrostatic loading. There are two possible plastic regimes corresponding respectively to the tensile and compressive stresses. For the associated case (dilation angle equal to the friction angle), the collapse is complete (the whole sphere is plastified) with a unique regime. For the non associated cases, we consider weaker solutions (partial collapse and regime change). Nevertheless, we show the collapse is complete and exhibits a single regime. Consequently, the collapse stress field and the limit load do not depend on the value of the dilation angle. This theoretical result is confirmed by numerical simulations.

In Gurson's footsteps, Guo and co-workers [2] proposed a macroscopic plastic model for porous solid with pressure-sensitive dilatant matrix obeying to the normality law (associated material). The aim of the second part is to extend the previous model to the non associated materials for which the dilatancy angle is not equal to the friction one. For such materials, the first author proposed a new modelling based on the concept of bipotential, a function of both dual variables, the plastic strain rate and stress tensors ([3], [4], [5]). On this ground, we use a variational approach to deduce a macroscopic model for porous materials.

References

- [1] Gurson, A.L., 1977. Continuum theory of ductile rupture by void nucleation and growth – part I: Yield criteria and flow rules for porous ductile media, *Journal of Engineering Materials and Technology*, 99, 2-15.
- [2] Guo, T.F., Faleskog, J., Shih, C.F. 2008. Continuum modeling of a porous solid with pressure-sensitive dilatant matrix. *Journal of the Mechanics and Physics of Solids*, 56, 2188-2212.
- [3] de Saxcé, G., Feng, Z.Q., 1991. New inequality and functional for contact friction: The implicit standard material approach. *Mechanics of Structures and Machines*, 19, 301-325, 1991.
- [4] de Saxcé, G., 1992. Une généralisation de l'inégalité de Fenchel et ses applications aux lois constitutives. *C. R. Acad. Sci. Paris, Sér. II*, 314, 125-129, 1992.
- [5] de Saxcé, G., Bousshine, L., 1998. Limit Analysis Theorems for the Implicit Standard Materials: Application to the Unilateral Contact with Dry Friction and the Non Associated Flow Rules in Soils and Rocks. *Int. J. Mech. Sci.*, 40(4), 387-398.

“Three-dimensional shakedown solutions and their application to road pavement design”

Juan Wang, Hai-Sui Yu

Nottingham Centre for Geomechanics, Faculty of Engineering, University of Nottingham, Nottingham, UK, NG7 2RD

(juan.wang@nottingham.ac.uk; hai-sui.yu@nottingham.ac.uk)

Abstract

A rigorous lower-bound solution is developed for shakedown limits of three-dimensional half-space under moving surface loads. Firstly, an analytical shakedown condition is derived to provide a maximum boundary to the lower-bound shakedown limit. Then, an optimisation procedure is used to search for the lower-bound shakedown limit that satisfies all Melan's lower-bound shakedown criteria. In particular, this procedure calculates a critical self-equilibrated residual stress field that also satisfies the yield criterion.

This lower-bound solution is then served as a design basis for flexible road pavements in the prevention of excessive plastic deformation. Finite-element-calculated elastic stress fields are used to derive the optimum shakedown multiplier for each pavement layer and the minimum shakedown multiplier is taken as the pavement shakedown load limit. The results of the analyses show significant influences of contact shape, surface frictional coefficient, material properties and layer thicknesses on the shakedown load limits. A new pavement design method is suggested by plotting thickness design charts using the shakedown solutions and choosing the thickness combination based on the design traffic load.

“Direct evaluation of the post-buckling behavior of slender structures through a numerical asymptotic formulation”

Giovanni Garcea, Antonio Bilotta, Raffaele Casciaro

Department of Engineering Modeling, University of Calabria, Italy *E-mail:* giovanni.garcea@unical.it,
antonio.bilotta@unical.it

Keywords: slender structures, post-buckling, asymptotic analysis.

Abstract

The analysis of slender structures characterized by complex buckling and postbuckling phenomena and by a strong imperfection sensitivity, is highly penalized by a lack of adequate computational tools. When the goal is the global evaluation of the structural collapse safety the analysis should consider all possible loadings, including the deviations due to load imperfections and geometrical defects. In this respect standard path-following approaches, aimed at recovering the equilibrium path for a single loading case and assigned imperfections, cannot be considered a satisfactory answer for this purpose. In fact, as the single analysis is computationally quite expensive, performing a complete investigation to consider all possible imperfection shapes is really expensive, if we do not have reliable information about the worst imperfection shapes.

The present lecture aims to provide an introduction to the nonlinear analysis of slender elastic structures through the use of the asymptotic approach. The latter essentially is a finite element implementation of Koiters nonlinear theory of elastic stability [1] and it constitutes a reliable tool for predicting the initial post-critical behavior in both cases of limit or bifurcation points (see [2, 3, 4] and reference therein). Its main advantage lies in the possibility of performing an efficient and robust imperfection sensitivity analysis even in the case of multiple, almost coincident, buckling loads. Moreover, it can provide a-priori information on the worst imperfection shapes for the structure, information which can be exploited for performing successive, more detailed, investigations through a specialized path-following analysis.

The fundamental idea of the asymptotic algorithm is to transform a snapping problem into a bifurcation one, to solve the latter and, finally, to reconstruct the solution of the snapping problem by using the asymptotic approach [2, 7]. The algorithm is based on the following steps: (*s1*) the fundamental path is obtained as a linear extrapolation starting from the initial known rest configuration; (*s2*) a bifurcation search is performed along the fundamental path obtaining the relevant buckling loads and associated modes; (*s3*) an extrapolation of the total potential energy starting from the first bifurcation point (or from an average point in the bifurcation cluster) is obtained through a consistent asymptotic expansion in terms of modal amplitudes (*s4*) the equilibrium path is finally obtained by solving a nonlinear system of equations in terms of ξ_i ; using a standard path-following method. The analysis is very fast, of the same order as a linearized stability analysis and new analyses relative to different imperfections only require step (*s4*) to be repeated so employing only a very small fraction of the main analysis time.

The FEM implementation of Koiters asymptotic approach is a tool which is still not so widely diffuse within computational mechanics libraries. This is essentially due to its high requirements with respect to the continuum model description and the relative finite element implementation. The following aspects require a careful formulation: (*i*) *the continuum structural model*; (*ii*) *the finite element description*; (*iii*) *the choice of variables used to describe the strain energy*. Regarding point (*i*) as the approach is based on a fourth-order expansion of the strain energy, a careful tuning of the structural model and of the related kinematical relationships, at least until the fourth-order strain energy variation, is necessary [4, 5]. Standard nonlinear structural beam and plate elements generally provide an inappropriate description of the third and fourth-order variations and can lead to noticeable errors in the results. With respect to point (*ii*) attention has to be paid to the finite element discretization in order to avoid interpolation locking phenomena that could completely destroy the accuracy of the solution. The use of a mixed format (in both displacement and stresses) is useful to eliminate the problem [2, 3]. Finally, with respect to point (*iii*), an appropriate choice of the variables used to formulate the problem equations is required in order to avoid a nonlinear extrapolation locking phenomenon as described in [6]. Once more, the use of a mixed format gives the best results.

References

- [1] W.T. Koiter, On the stability of elastic equilibrium, 1945. Thesis, Delft. English transl. NASA TT-F10 (1967) 883 and (1970) AFFDL/TR70-25.

- [2] R. Casciaro, Computational Asymptotic Post-Buckling Analysis of Slender Elastic Structures, in Phenomenological and Mathematical Modelling, in Structural Instabilities, Pignataro M., Gioncu V. eds. (2005) Springer Verlag.
- [3] A. D. Lanzo, G. Garcea, Koiter's analysis of thin-walled structures by a finite element approach, International Journal for Numerical Methods in Engineering, 39, (1996), 3007-3031.
- [4] G. Garcea, A. Madeo, G. Zagari, R. Casciaro, Asymptotic post-buckling FEM analysis using corotational formulation, International Journal of Solids & Structures, 46,(2009), 377-397.
- [5] G. Garcea, A. Madeo, R. Casciaro, Derivation of geometrically exact nonlinear models for beams and plates using linear solutions, Submitted to Computers & Structures.
- [6] G. Garcea, G. Salerno, R. Casciaro, Extrapolation locking and its sanitization in Koiter's asymptotic analysis, Computer Methods in Applied Mechanics and Engineering, 180, 1-2, (1999), 137-167.
- [7] F. Brezzi, M. Cornalba, A. Di Carlo, How to get around a simple quadratic fold, Numer. Math., 48, (1986), 417-427.

“Shakedown analysis of steel structures under ellipsoidal variable load domain”

By M.-A. A. Skordeli and C.D. Bisbos

Laboratory of Metal Structures,
Dept. of Civil Engineering, Aristotle University,
GR-54124 Thessaloniki, Greece
Email: cbisbos@civil.auth.gr

Abstract

Shakedown analysis (SDA) is based on the fundamental works of Melan (lower bound approach) and Koiter (upper bound approach). Its computational treatment is usually based on the coupling of the finite element method with mathematical optimization techniques. A frequently used computational model is based on the quasi-lower bound formulation, where the formal scheme of lower bound approach is retained, but the finite element data are obtained via the standard – and today’s dominant - displacement finite element method. In this formulation, the linear or nonlinear inequalities, which appear in the mathematical optimization problem, correspond to the fulfillment of the yield criteria at each check point (Gauss point of the FEM mesh).

Computational SDA techniques concern, almost exclusively, polygonal and bounded variable load domains (known as polytopic domains in the mathematical programming language). The boundary of this variable load domain type is defined by hyper planes and, by convexity, it is sufficient that the local yield criteria are fulfilled only at the vertices of the variable load domain. This way, the resulting number of inequality constraints in the aforementioned quasi-lower bound formulation is finite, i.e. the arising mathematical optimization problem is an ordinary one.

Situation alters drastically if the variable load domain is nonlinear. In this case, the local yield criteria must be fulfilled at each point of the curved boundary of the load domain and the number of inequality constraints becomes infinite, i.e. the mathematical programming problem ceases to be an ordinary one and now becomes a construct known as a semi-infinite programming (SIP) problem, which is very hard to address. In general, SIP problems are considered as computationally intractable.

To the author’s knowledge, research work concerning nonlinear variable load domains is very rare. Pycko has developed a cycle-oriented incremental analysis of SDA problems with nonlinear variable load domains [1]. Bisbos and Ampatzis have used parameterization of the nonlinear load domain in order to develop a computational bi-level methodology, capable to solve respective SDA problems [2]. The assumption that the local yield criteria are piecewise linear is crucial in their work.

The present work, based on the doctoral thesis of the first author [3], concerns computational SDA of steel structures with nonlinear yield criteria under ellipsoidal variable load domain. Basically, ellipsoidal yield criteria are considered, which are of great importance to steel structures, as the von Mises, Hill and Ilyushin yield criteria belong to this category.

The development of the computational SDA technique with ellipsoidal load domain and ellipsoidal yield criteria is based on robust optimization techniques [4]. Robust optimization, an emerging branch of convex mathematical programming, addresses computationally intractable problems and their formulation as tractable nonlinear programming problems. It deals with ellipsoidal uncertainties as well as worst case scenarios. As a result, robust optimization techniques are exploited in order to make computationally tractable the shakedown analysis problems with ellipsoidal yield criteria under ellipsoidal load domain. This way, a semi-definite programming (SDP) problem arises. Remarkably, SDP problems are the natural generalizations of second-order conic programming (SOCP) problems, which describe the quasi-lower bound approach to the SDA problems of steel structures with the von Mises criterion under standard polytopic variable load domain.

Furthermore, having the aforementioned SDP technique at hand, a respective approximate technique is proposed for steel structures with general, non-ellipsoidal yield criteria. The maximal inscribed and the minimal circumscribed ellipsoids to common steel section yield criteria - presented by the authors in a previous

publication [5]- are used in order to obtain numerical upper and lower bounds to the SDA safety factor, corresponding to the quasi-lower bound problem at hand. Examples are presented and several issues are discussed.

References

- [1] S. Pycko (1997) A cycle-oriented incremental analysis of shakedown problems. International Journal of Numerical Methods Eng, Vol. 40, pp. 3163–3179.
- [2] C.D.Bisbos and A.T.Ampatzis (2008) Shakedown analysis of spatial frames with parameterized load domain, Engineering Structures, Vol. 30, pp. 3119-3128.
- [3] M.-A.A. Skordeli (2010) Shakedown analysis of metal structures, subjected to ellipsoidal variable repeated loading, via robust optimization techniques, Doctoral Dissertation, Dept. of Civil Engineering, Aristotle University of Thessaloniki, Greece.
- [4] A. Ben-Tal and A. Nemirovski (2001) Lectures on modern convex optimization. Analysis, Algorithms and Engineering Applications. SIAM, Philadelphia.
- [5] M.-A. A. Skordeli and C.D.Bisbos (2010) Limit and shakedown analysis of 3D steel frames via approximate ellipsoidal yield surfaces, Engineering Structures, Vol. 32, pp. 1556-1567.

“Discontinuity Layout Optimization (DLO)”

Matthew Gilbert¹, Colin C. Smith¹, Samuel J. Hawksbee¹

¹ The University of Sheffield, Dept Civil & Structural Engineering, Mappin St, Sheffield S1 3JD
m.gilbert@sheffield.ac.uk; c.c.smith@sheffield.ac.uk; cip09sjh@sheffield.ac.uk; <http://cmd.shef.ac.uk>

Keywords: Plasticity, limit analysis; discontinuity layout optimization; bearing capacity, notched plate.

Abstract

Discontinuity Layout Optimization (DLO) is a recently developed numerical limit analysis procedure which makes use of a novel discontinuous approach. Key benefits of the method include (i) solutions in the form of familiar sliding block mechanisms, and (ii) the ability to handle singularities in the stress or displacement fields naturally, in contrast to traditional finite element limit analysis procedures where solutions can be sensitive to mesh arrangement around singularities.

DLO procedure (translational mechanisms)

Stages in the DLO formulation for plane strain translational problems can be illustrated by considering a simple example problem. Thus in Figure 1 a surcharge is applied to a block of soil close to a vertical cut. The area occupied by the block of soil is first populated with nodes as shown in Figure 1(b). Each pair of nodes is then interconnected with potential linear discontinuities, as shown in Figure 1(c). The critical failure mechanism for the given nodal discretization is then identified from the set of potential discontinuities using a linear programming optimization procedure (Smith & Gilbert, 2007), Figure 1(d).

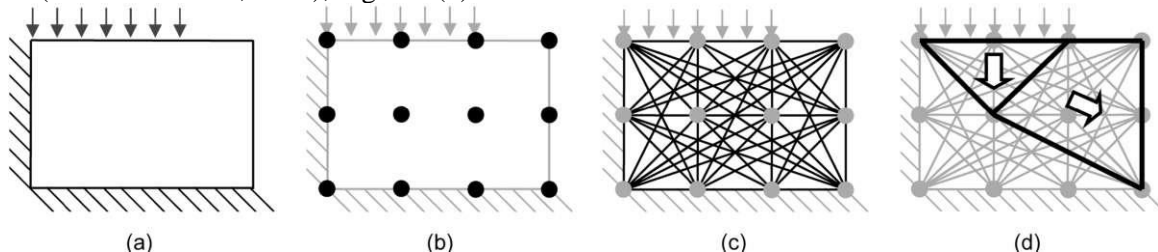


Figure 1: Stages in DLO procedure: (a) starting problem (surcharge applied to block of soil close to a vertical cut); (b) discretization of soil using nodes; (c) interconnection of nodes with potential discontinuities; (d) identification of critical subset of potential discontinuities using optimization (giving the layout of slip-lines in the critical failure mechanism) (after Gilbert et al., 2010).

The mechanism identified consists of rigid polygons bounded by linear discontinuities along which changes in velocity are permitted, as illustrated in Figure 1(d). This basic DLO procedure has been implemented in a short («100 line) MATLAB script for analysing cohesive-frictional plane strain problems involving rectangular domains, and this can be downloaded from <http://cmd.shef.ac.uk/dio>.

Extension to rotational mechanisms using a hybrid rotational/translational method was outlined by Gilbert et al. (2010). This technique has been implemented in a commercially available software package LimitState:GEO. Sample output from the software is provided in Figure 2 for the Prandtl problem. This solution was obtained in a few seconds on a desktop PC, and is within 1% of the known exact solution of $2 + n$. In addition to its primary purpose as a tool for use in industry, the highly visual nature of the output also makes the software useful in teaching, and the software is freely available for academic teaching and research use, see <http://www.iimitstate.com/education>.

Rotational mechanisms

Modification of the basic procedure outlined above to use curved as well as linear potential discontinuities, allows fully rotational problems to be analysed to a high degree of accuracy. An sample solution to the well known 'notched plate' problem (half width notch) is illustrated in Figure 3(a). With a coarse 16 x 16 nodal distribution, a solution of 1.13227 was obtained, which compares very favourably to the limit analysis solution of 1.13582 (or 1.13156, extrapolated) obtained by Christiansen and Andersen (1999).

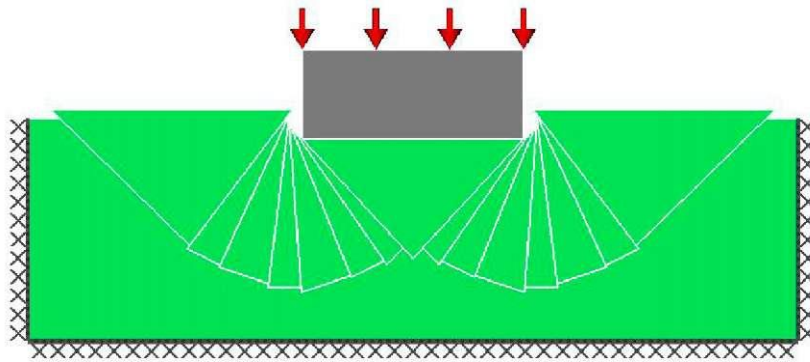


Figure 2: Sample DLO solution to the Prandtl problem obtained using LimitState:GEO. Solution of 5.175 obtained using a nodal refinement of approx. 1000 nodes. Singularities at punch edges are handled inherently by the method.

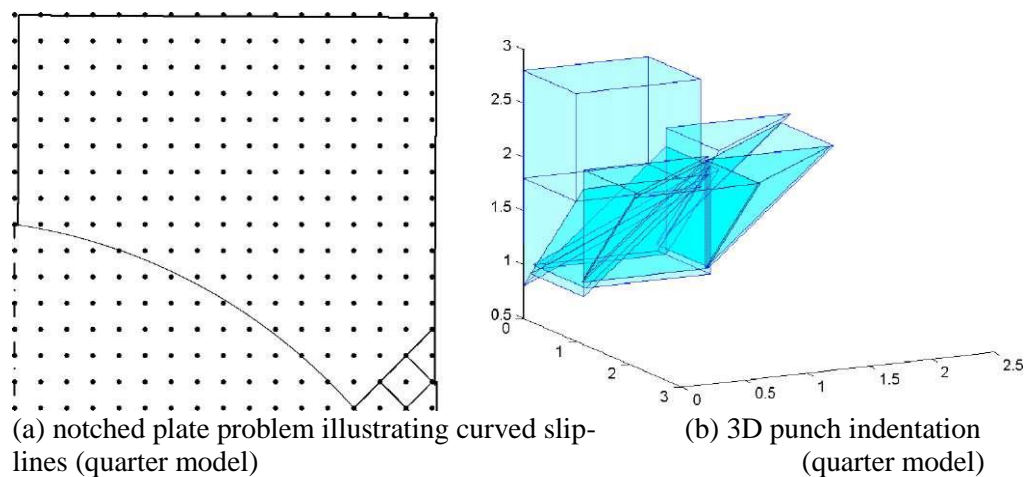


Figure 3: Examples of (a) rotational and (b) 3D DLO analysis solutions

Three Dimensional Analysis

The basic DLO formulation may be modified to allow three-dimensional analysis problems to be tackled. The formulation involves the use of polygonal potential discontinuities connecting any set of co-planar nodes, along which velocity jumps are permitted. Compatibility is enforced along the edges of the polygonal discontinuities and the minimum collapse load for a given nodal discretization, together with the corresponding collapse mechanism, can be obtained using efficient second order cone programming algorithms. The mechanism produced comprises rigid polyhedra bounded by the polygonal discontinuities.

The analysis of the 3D square punch indentation problem is illustrated in Fig 3(b). Despite use of a coarse nodal distribution, a bearing capacity factor equal to 6.424 was obtained, which compares well with the best upper bound solution in the literature of 6.22, obtained by Salgado et al. (2004) using finite element limit analysis.

References

- [1] Christiansen E, and Andersen KD (1999) Computation of collapse states with Von Mises type yield condition. *Int. J. Numer. Meth. Engng.* 46, 1185-1202.
- [2] Gilbert M, Smith CC and Pritchard TJ (2010) Masonry arch analysis using discontinuity layout optimisation. *Proceedings of the Institution of Civil Engineers - Engineering and Computational Mechanics*, 163 (3), September, pp 155-166.
- [3] Gilbert M, Smith CC, Haslam I, and Pritchard T (2010). Application of discontinuity layout optimization to geotechnical limit analysis. *Proc. 7th Eur. Conf. Num. Meth. Geotech. Eng.*
- [4] Salgado R, Lyamin AV, Sloan SW, and Yu HS. (2004) Two and three-dimensional bearing capacity of foundations on clay. *Geotechnique*, 54(5), 297-306.
- [5] Smith C and Gilbert M (2007) Application of discontinuity layout optimization to plane plasticity problem. *Proc. Roy. Soc. A.* 463(2086), 2461-2484.

“A Quasi-Periodic Approximation based Model Reduction for Limit Analysis of Micropile Groups”

Zied Kammoun¹, Hichem Smaoui^{1,2}, Joseph Pastor³

¹ LAMOED Laboratory, ENIT, University of Tunis El Manar, Tunisia

² College of Engineering, Al-Kharj University, Saudi Arabia

³ LOCIE Laboratory, Polytech'Savoie, University of Savoie, France

Keywords : Limit analysis, Static Approach, Pile Groups, Micropile, Quasi-periodic, Finite Elements.

Abstract

Micropiles are used in soil reinforcement and foundation works beneath existing buildings. Micropile technique was initially developed as early as 1952 [1]. However, the behaviour of micropile groups is still not well understood, particularly because of the complex geometry of large soil-micropile systems that challenged the development of modeling methods. Various approaches are used for predicting the bearing capacity of micropile groups. Simplified analytical methods are commonly used in engineering practice whereas elastoplastic analysis is often applied in special applications and in research. Another alternative is limit analysis by direct methods. The merit of Limit Analysis (LA) is the rigorous underlying theoretical basis and the high level of accuracy that may be reached. However, reasonably accurate prediction for micropile groups of practical size requires finely discretized finite element models leading to numerical optimization problems that are too large to be directly tractable by available algorithms.

In an attempt to circumvent the problem size difficulty, an overlapping domain decomposition approach has been developed [2,3,4] that converts the original numerical LA problem into a sequence of smaller LA like subproblems that are solved iteratively. This approach has proven to be successful in solving problems that are untractable when solved directly. In the present work, another technique is presented that aims at reducing the size of the numerical LA problem for uniformly spaced micropile groups. It is inspired from the case of fiber reinforced composites which is suited to modeling using periodic homogenization [4,5]. Considering a plane strain representation, the proposed method takes advantage of the periodicity of the pile-soil system configuration in the interior of the reinforced zone which is assumed to exhibit a periodic behaviour. A representative volume element (RVE) is constituted by a micropile and half the width of soil on each side in addition to the underlying soil volume. Regardless of the number of micropiles it includes, the periodic zone, denoted Zone I, is replaced by a single periodic representative volume element (PRVE) fulfilling built-in periodicity and inter-RVE continuity constraints. This results in a considerable reduction in problem size at the cost of an approximation error. Interestingly, the error is on the conservative side, preserving the lower bound nature of the solution of the static problem and the upper bound character of the kinematic solution. Edge zones, separating the periodic zone from the natural soil, are defined by a few RVE's on each side and labelled as Zone II. The finite element mesh corresponding to these edge zones and the natural soil (Zone III) remains unchanged. Furthermore, the detailed modeling of the soil-micropile composite at the RVE level, both in the horizontal and vertical directions, has the merit of accounting for the toe and lateral effects on the bearing capacity as well as the interaction effect between micropiles, made possible by the full consideration of the soil layers underlying the reinforced zone.

The LA problem considered is that of a Tresca soil reinforced by a micropile group supporting a weightless foundation of width b loaded by a force F . The soil cohesion is $c=10$ kPa and depth $H=30m$. The micropile length is $h=20m$ and width $d=0.2m$. The bearing capacity of the foundation is determined as the maximum load F that, together with a stress field σ , form a statically and plastically admissible pair. The associated numerical optimization problem is denoted P_0 .

In the reduced problem, denoted P , Zone I is modeled using a single PRVE and the load is defined as the sum of the loads supported by the RVEs in Zone II and the load supported by the PRVE scaled up by the number of micropiles belonging to Zone I. Since the solution for the reduced problem is admissible for P_0 , it provides a lower bound for the original LA problem.

Two cases of load distribution between the soil surface and the micropile tips are considered. In the first, the foundation is assumed to be supported solely by the micropiles. In the second, it is supposed to rest on both the soil surface and the micropile tips. In both cases the kinematic and static bounds of the bearing capacity are determined by solving the direct problem. Furthermore, a static bound is estimated by solving the reduced problem resulting from the quasi-periodic approximation. Results are produced for a range of micropile spacings

to assess the effects of spacing and load transmission mode.

The load capacity increases with spacing in general. The exception is when spacing exceeds a threshold of $6.8m$ and the applied load is transmitted via the micropiles only. Under these conditions the bearing capacity is nearly indifferent to spacing.

The error on the lower bound resulting from the quasi-periodic approximation is small from a practical viewpoint. For the case of a load supported by the micropiles only, the relative error reaches 4.2% in the worst case over the range of spacings considered and tends to decrease as the spacing increases. When the applied load is supported by both the micropiles and the soil, the relative error reached 9% in the worst case and appeared to increase with spacing. These opposite trends can be explained from examination of the failure mechanisms. In the case when the load is supported exclusively by the micropiles, these inclusions tend to behave in the same pattern as the spacing increases, justifying the periodic assumption. This pattern is characterized by a localization of the failure zone in a thin layer of soil surrounding the micropile. When the load is distributed between the micropiles and the soil surface the failure zone continues to expand over the volume of soil as the spacing increases and the failure mechanism maintains a global character that explains the persistence of approximation error due to the quasi-periodic reduction.

References

- [1] Lizzi F. *The 'pali radice' (root piles) - A state of the art report*. Symposium on recent developments in ground improvement techniques. Bangkok, Thailand. 1982
- [2] Zied Kammoun, Franck Pastor, Hichem Smaoui and Joseph Pastor : Large static problem in numerical limit analysis: A decomposition approach. *Int. J. Num. Anal. Methods in Geomech.* Volume 34, Issue 18, pages 1960–1980, 25 December 2010.
- [3] Zied Kammoun, Joseph Pastor and Hichem Smaoui : «Limit analysis of a soil reinforced by micropile group: a decomposition approach» Second International Workshop on Direct Methods Ecole Centrale de Lille , France, 22-23 oct. 2009
- [4] F. Pastor, E. Loute, J. Pastor. - *Limit analysis and convex programming: a decomposition approach of the kinematical mixed method*, *Int. J. Numer. Meth. Engng.*, Vol. 78, 254-274, 2009.
- [5] Francescato P., Pastor J. - *Limit Analysis and Homogenization: predicting limit loads of periodic heterogeneous materials* - *Eur. J. Mech. -A/Solids*, vol. 16, pp. 235-253, 1997.

“A Ratchetting Problem in Materials Science”

Alan R. S. Ponter

Department of Engineering, University of Leicester, UK,
asp@le.ac.uk

Abstract

There are a class of problems in polycrystalline metals and alloys where the microstructure has components with differing thermal expansion properties. Such materials include polycrystals where each crystal exhibits anisotropic thermal expansion, e.g. alpha-uranium and zinc. Thermal cycling through the martensitic phase change temperature in steels has an equivalent effect. Metal matrix composites have constituents, for example aluminium and a ceramic, with differing coefficients of thermal expansion. Such materials can show rapid plastic and creep ratchetting when subjected to variable temperature and small applied stress. The mechanisms for this form of ratchetting provide an interesting problem in plasticity and solutions are of interest to material science.

In the literature, the theory widely used to describe this behaviour derives from solutions produced by Anderson and Bishop [1-3] in the 1950's as a way of understanding the ratchetting properties of irradiated alpha-uranium. The solution method was based upon that of the classic paper by Bishop and Hills [4], which derived the Maximum Work Principle for polycrystalline metals. The Anderson and Bishop solutions were published in a somewhat condensed form [3] in a conference proceeding now difficult to obtain. Original technical reports [1,2] were not published and were probably subjected to security conditions. Greenwood and Johnson [5] adopted their solutions for the martensitic phase change problem, where again a summary was included as an appendix. The solutions have been republished many times and appear in standard texts.

The Anderson and Bishop [1-3] solutions are simple and mainly kinematic, i.e. the emphasis is on compatibility and conditions of equilibrium are not consistently imposed. The presentation will resolve the problem, taking into account our current methods understanding of ratchetting. It is shown that the Anderson and Bishop [1-3] solutions are only correct for a very restricted class of problems, described correctly in the restricted UKAEA [1,2]. New solutions are derived and the conditions when the Anderson and Bishop solutions apply is discussed.

References

- 1) Anderson R.G. and Bishop J.F.W, The effect of irradiation and thermal cycling of the deformation of alpha-uranium, Report IGR-TN/C.681, United Kingdom Atomic Energy Authority, October 1957.
- 2) Anderson R.G., The effect of small thermal cycles on the creep strength of alpha-uranium under small stresses, Report IGR-TN/C.854, United Kingdom Atomic Energy Authority, 1958.
- 3) Anderson R.G. and Bishop J.F.W, The effects of neutron irradiation and thermal cycling on the permanent deformation in uranium under load, Institute of Metals Symposium on Uranium and Graphite, Paper 8, 1962.
- 4) Bishop, J.F.W. and Hill, R , A theory of the plastic distortion of a polycrystalline aggregate under combined stresses. Phil Mag., 42, p414, ibid p.1298.
- 5) Greenwood G.W. and Johnson R.H., Proc. Roy. Soc, Lon., 283A, 403-433, 1965.

“Evaluation of the peak load corresponding to pre-assigned design criteria in composite laminates by the Linear Matching Method”

O. Barrera¹, A.C.F. Cocks¹ and A.R.S. Ponter²

¹Department of Engineering Science University of Oxford, Parks Road, Oxford OX1 3PJ, UK

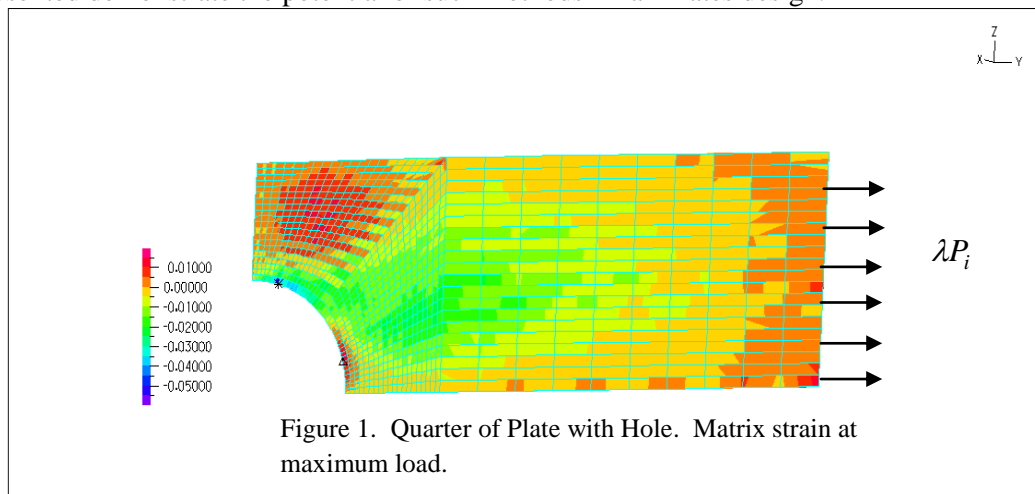
²Department of Engineering University of Leicester, University Road, Leicester LE1 7RH, UK

Abstract

The paper deals with the development of a Direct Method [1] for the evaluation of the maximum load constraints on the nonlinear behaviour of the matrix and fibres in a laminate structure. This is achieved by adopting a consistent micro-macro model for linear behaviour with an extension of the Linear Matching Method (LMM), previously extensively applied to Direct Methods in plasticity. Such Direct Methods may be applied to an elastic-perfectly plastic model of laminate behaviour (see [2, 3]) with success. However, depending on the mode of laminate failure, assuming elastic-perfect plastic behaviour is not always appropriate.

The Linear Matching Method (LMM) [3,4] consists, essentially, of an iterative programming method that allows a direct evaluation of the load corresponding to predefined kinematic restraints. The primary area of application has been the evaluation of limits appropriate for metallic structures particularly those subjected to severe thermo-mechanical loads. Methods have been developed for the life assessment of power plant at high temperature and an entire set of such methods now form part of the UK Life Assessment Method R5 [6, 7]. The work presented in this paper is a first attempt to develop an appropriate method of this type for composite materials.

We explore the possibility of developing a method whereby the maximum permissible load may be related directly to a particular strain condition or failure mode at any point in the structure, in any layer of the laminate and at any point in the microstructure, taking into account nonlinear behaviour of the composite material constituents. We combine a kinematically consistent micromechanics model with significant elements of classical laminate theory [12] and a simple description of micro failure. This first step then allows the methodology of the Linear Matching Method to be explored for this class of new problems in its simplest form. The problem deals with a structure of surface S and volume V subjected to a load distribution λP_i where P_i is a chosen load distribution on S_T , part of the surface, and λ is a scalar multiplier. The objective of the method is to evaluate the value of λ so that the strain field in the microstructure corresponds to a prescribed design condition, while all other conditions of the continuum problem are satisfied: equilibrium, compatibility and consistency with the material behaviour. The design condition relates to failure of either the matrix or the fibres. The paper explores the theoretical requirements for such a method for laminates, building upon previous work for portal frames where softening behaviour is allowed [9-10]. Numerical applications regard a Glass-Epoxy plate in tension (Figure 1), containing a circular hole, where elastic-brittle behaviour for the fibres and non linear behaviour of the matrix are considered [11]. The dominating design criterion chosen is a limiting inelastic strain in the matrix. The significance of this example is its ability to show the usefulness of the procedure to deal with the effect of the ductility of the matrix on the maximum load. Despite the simplicity of both the model and the examples, the solutions presented demonstrate the potential of such methods in laminates design.



References

- 1) Maier, G., Pastor, J., Ponter, A.R.S. and Weichert, D., "Direct Methods for Limit and Shakedown Analysis", In: Comprehensive Structural Integrity – Vol.3 Ed. R de Borst & H.A. Mang, Elsevier Science, Oxford.
- 2) A. A. Pisano, P. Fuschi, "A numerical approach for limit analysis of orthotropic composite laminates", *International Journal for Numerical Methods in Engineering*, 70, 71-93, 2007.
- 3) P. Fuschi, A. A. Pisano, O. Barrera. "Limit analysis of orthotropic laminates by Linear Matching Method", in: D. Weichert, A.R.S. Ponter (Eds.), *Limit States of Materials and Structures- Direct Methods*, Springer, Wien, 197-220, 2009.
- 4) Ponter A.R.S. and Engelhardt M., "Shakedown Limits for a General Yield Condition: Implementation and Examples for a von Mises Yield Condition", *European Journal of Mechanics, A/Solids*, 19(3): 423-446, 2000.
- 5) Ponter A.R.S., Fuschi P. and Engelhardt M., "Limit Analysis for a General Class of Yield Conditions", *European Journal of Mechanics, A/Solids*, 19(3), 401-422, 2000.
- 6) Chen H. F., Ponter A. R. S. and Ainsworth R. A., (2006), "The linear matching method applied to the high temperature life assessment of structures. Part 1. Assessments involving constant residual stress fields", *Int. Jn. of Pressure Vessels and Piping*, 83, 123-135.
- 7) Chen H. F., Ponter A. R. S. and Ainsworth R. A., (2006), "The linear matching method applied to the high temperature life assessment of structures. Part 2. Assessments beyond shakedown involving changing residual stress fields", *Int. Jn. of Pressure Vessels and Piping*, 83, 136-147
- 8) Barrera O., Cocks A. C. F. and Ponter A. R. S. "Evaluation of the Convergence Properties of the Linear Matching Method for Computing the Collapse of Structural Components", *European Journal of Mechanics A/Solids*, 28: 655–667, 2009.
- 9) O. Barrera, A.R.S. Ponter, and A.C.F. Cocks, "Application of the Linear Matching Method to materials that exhibit softening", XX conference AIMETA, Ancona, 2009.
- 10) O. Barrera, A.R.S. Ponter, and A.C.F. Cocks, "Extension of the Linear Matching Method applied to frame structures made of a material that exhibit softening", *European Journal of Mechanics A/Solids* 30 (2011) 783-793.
- 11) O. Barrera , A.C.F. Cocks , A.R.S. Ponter (2011), The Linear Matching Method applied to composite materials: a micromechanical approach, *Composites Science and Technology*, 71, 797–804.
- 12) Hull D. and Clyne T.W., "An Introduction to Composite Materials", Cambridge University Press, 2nd Ed., 1996.

“Recent development and application of the Linear Matching Method for design limits in plasticity: an overview”

Haofeng Chen

Department of Mechanical and Aerospace Engineering,
University of Strathclyde, Glasgow, G1 1XJ, UK

Abstract

Engineering design and integrity assessment of components under the action of cyclic thermal and mechanical loading require the assessment of load histories for which certain types of material failure do not occur [1]. This involves the determination of the shakedown limit, ratchet limits and plastic strain range concerning fatigue crack initiation in a low cycle fatigue assessment.

In this paper a state-of-the-art direct method, the Linear Matching Method (LMM), is presented for the evaluation of the shakedown limit, ratchet limits and plastic strain range concerning fatigue crack initiation for an elastic-perfectly plastic body subjected to cyclic thermal and mechanical load history. These design limits in plasticity have been solved by characterizing the steady cyclic state using a general cyclic minimum theorem. For a prescribed class of kinematically admissible inelastic strain rate histories, the minimum of the functional for these design limits are found by either global minimization process or dual minimization process.

In this research, the fundamentals of numerical methods for design limits in plasticity are readdressed with three objectives in mind. The first is to provide a more general and unified LMM approach for wider class of problems and potential procedures for both upper and lower bound design limits. The second is to investigate and improve the convergence issues in the iterative approach. The third objective is to verify the efficiency and effective of the LMM on the assessment of design limits in plasticity by applying it to three distinctive practical problems.

The strategy of locating shakedown and ratchet limits consists of defining an appropriate class of kinematically admissible strain rate histories $\dot{\epsilon}_{ij}^c$ then solving a corresponding minimizing process for $I(\dot{\epsilon}_{ij}^c, \lambda)$ by considering the incremental form;

$$I(\dot{\epsilon}_{ij}^c, \lambda) = \sum_{n=1}^N I^n, I^n(\Delta \epsilon_{ij}^n, \lambda) = \int_V \left\{ \sigma_{ij}^n \Delta \epsilon_{ij}^n - (\lambda \hat{\sigma}_{ij}(t_n) + \rho_{ij}(t_n) + \bar{\rho}_{ij}) \Delta \epsilon_{ij}^n \right\} dV, \rho_{ij}(t_n) = \sum_{l=1}^n \Delta \rho_{ij}(t_l) \quad (1)$$

where $\dot{\epsilon}_{ij}^c$ is replaced by a sequence of increments of strain $\Delta \epsilon_{ij}^n$ occurring at a sequence of N times t_n , $n=1$ to N, during the cycle. The shakedown analysis involves a global minimization of $I(\dot{\epsilon}_{ij}^c, \lambda)$ which makes use of the compatibility of the sum of the increments of plastic strain over the cycle. The calculation of the ratchet limit includes dual minimization processes, the first an incremental minimization for the evaluation of a cyclic history of residual stresses and plastic strain range in a stable cycle and the second a global minimization for the ratchet limit due to an extra constant load. The incremental minimization of $I^n(\Delta \epsilon_{ij}^n, \lambda)$ assumes the prior history of the residual stress is known and compatibility of the total elastic and plastic strain in the increment is used.

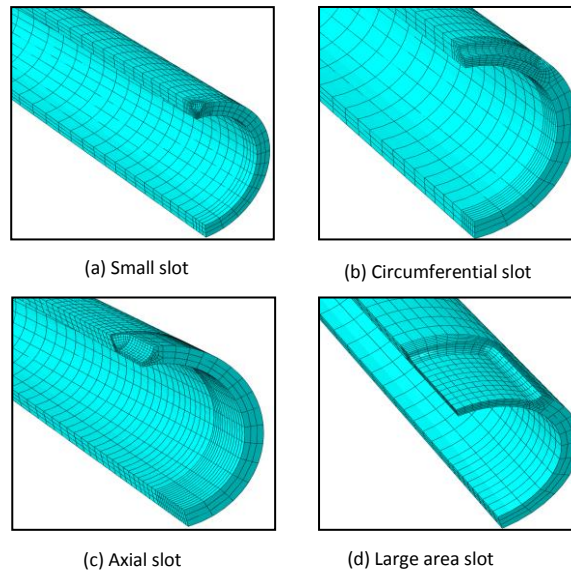


Figure 1: Example 1 of a pipeline with part-through slot: (a) small slot; (b) circumferential slot; (c) axial slot and (d) large area slot

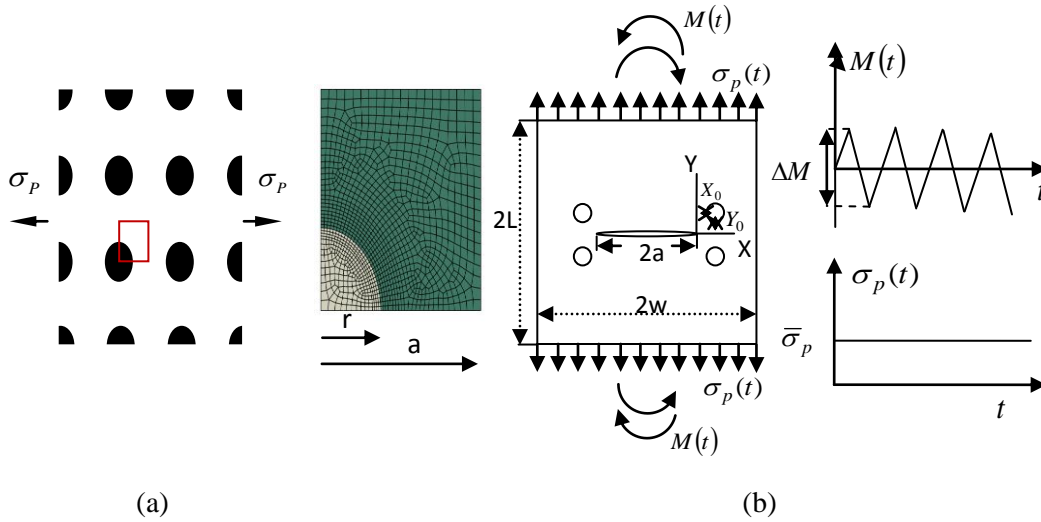


Figure 2: (a) Example 2 of fiber reinforced MMC ($V_f=11\%$) as loaded and the unit cell used in the FEA; (b) Example 3 of centre cracked plate with symmetric holes subjected to reversed bending moment range ΔM and constant tension $\bar{\sigma}_p$

The application of the LMM to three practical problems shown in Figs 1 and 2 confirms the efficiency and effectiveness of the method and demonstrates that Direct Methods may be applied to a much wider range of circumstances than have hitherto been possible.

Reference

[1] Ainsworth R.A. (editor) (2003): An assessment Procedure for the High Temperature Response of Structures. R5 Issue 3, British Energy Generation Ltd.

“Cone based Limit State Analysis of structures with non-conical yield criteria”

S. Krabbenhøft and L. Damkilde

Department of Civil Engineering Aalborg University, Aalborg, Denmark
K. Krabbenhøft

Centre for Geotechnical and Materials Modelling University of Newcastle, Australia

Abstract

Summary The paper deals with limit state problems formulated as lower-bound problems where the variables are stress parameters in each element. The problem results in an optimization problem where the objective is a linear function of the stress parameters i.e. load bearing capacity. The constraints are the yield criteria which in general are non-linear in the stress parameters and the equilibrium constraints which are linear. There exist very effective non-linear optimization codes for yield criteria which can be formulated as conical restrictions. However, for many practical applications the yield criteria cannot be formulated as conical restrictions. The paper propose an iterative method which in each step approximate the yield surface with a conical constraint. The method has been applied on a geotechnical problem with very good results.

Introduction

The pioneering work on computer methods for limit state analysis dates back to 1971, see [1]. The formulation was based on the finite element concept using the lower-bound method. The yield criteria were linearized securing that the optimization problem resulted in a standard linear programming problem. The problems which could be handled were relative modest in size and different methods were proposed in order to improve the efficiency, see e.g. [2] where the number of stress parameters were reduced by static elimination. Later there were different approaches in order to handle non-linear yield criteria, see e.g. [3] which proposed a subsequent linearization of the yield criteria. Interior point methods became increasingly popular compared to traditional linear programming algorithms. The interior point method was extended to non-linear yield criteria, see e.g. [4]. Another major change in the non-linear optimization was introduced by the so-called cone-based method, see e.g. [5]. The conical methods are very effective for non-linear yield criteria which can be formulated as conical restrictions.

Problem formulation

There exist a number of yield criteria which cannot be formulated in conical restrictions e.g. within geotechnics, where non-linear Mohr criteria can be applied, and within rock mechanics, where an often used yield criteria is Hoek-Brown's criteria. Material optimization e.g. layout of reinforcement in slabs can also be formulated as a limit state problem, see e.g. [6]. The yield criteria will include design variables which not always can be cast as conical restrictions.

The proposed method uses somewhat the same idea as was applied in [3] namely a subsequent approximation of the yield surface. Instead of a linearization as in [3] the yield surface is approximated with conical restrictions. Each iteration step is solved by means of the very effective conical program MOSEK, see [7]. After each iteration the yield criteria in all the points, where the yield criteria are checked, are updated based on the current stress level and approximated with a new conical restriction. The method has shown to be quite effective.

Example

The proposed method has been applied on an eccentrically loaded footing in plane strain, and Figure 1 shows the collapse form. The yield criteria is a non-linear Mohr envelope i.e. the increase in shear capacity is larger for small normal stress levels than for larger normal stress levels.

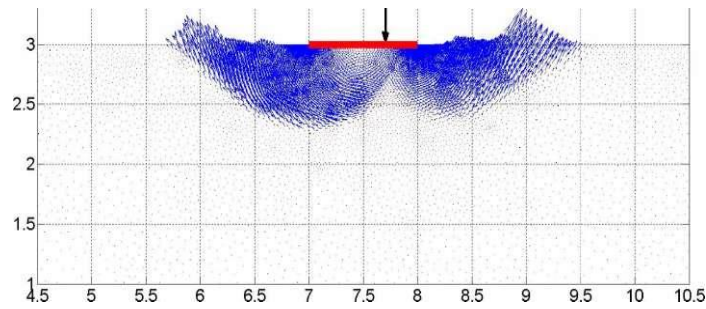


Figure 1: Collapse mode for eccentrically loaded footing

The structure is discretized by triangular elements, see [8], with a linear stress variation. The static constraints are the interelement stress continuities and equilibrium equations for each element. The collapse form shown in Figure 1 is illustrated by means of the so-called shadowprices on the equilibrium constraints.

The convergence of the method is very fast and is reached within approximately 10 global iterations. The method has not yet been tested on 3-D problems, but it is believed that convergence also will be reached in these cases.

References

- [1] D. E. Grierson and G. M. L. Gladwell. Collapse load analysis using linear programming. *Journal of the Structural Engineering Division, ASCE*, **97**, pp. 1561-1573, 1971.
- [2] L. Damkilde and O. Høyer. An efficient implementation of limit state calculations based on lower-bound solutions. *Computers & Structures*, **49**, pp. 953-962, 1993.
- [3] K. Krabbenhøft and L. Damkilde. Lower bound limit analysis of slabs with nonlinear yield criteria. *Computers & Structures*, **80**, pp. 2043-2057, 2002.
- [4] K. Krabbenhøft and L. Damkilde. A general non-linear optimization algorithm for lower bound limit analysis. *International Journal for Numerical Methods in Engineering*, **56**, pp. 165-184, 2003.
- [5] E.D. Andersen, C. Roos and T. Terlaky. On implementing a primal-dual interior-point method for conic quadratic optimization. *Mathematical Programming*, **95**, pp. 249-277, 2003.
- [6] S. Krenk and L. Damkilde. Limit Analysis and Optimal Design of Plates with equilibrium elements *Journal of the Structural Engineering Division, ASCE*, **120**, pp. 1237-1254, 1994.
- [7] Mosek. Optimization toolbox for MATLAB. Version 6.0 (Revision 93). 2001
- [8] P.N. Poulsen and L. Damkilde. Limit State Analysis of Reinforced Concrete Plates subjected to In-plane Forces. *International Journal of Solids and Structures*, **37**, pp. 6011-6029, 2000.