CENTRALE LILLE

Direct Methods 2009

Second International Workshop on Direct Methods

Ecole Centrale de Lille

22-23 october 2009

ABSTRACTS

LABORATOIRE de MECANIQUE de LILLE UMR CNRS 8107

E.C. LILLE
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U.S.T.L.

Second International Workshop on Direct Methods

ECOLE CENTRALE DE LILLE

Final schedule

Tuesday, 22th October – Hall *Poirier*

8:30 Welcome at the Ecole Centrale de Lille (see map for the access)

9:00 Introduction by Géry de Saxcé

9:10 A. R. S. Ponter
«*Direct Methods for the Evaluation of the Ratchet Limit for Perfect Plasticity*»

9:50 G. Garcea, L. Leonetti
«*On the effectiveness of numerical algorithms for the evaluation of the shakedown and limit loads* »

10:30 Coffee break, Room B16

10:45 A. Constantinescu, H. Maïtournam
"The role of elastoplastic material models in fretting-fatigue: shakedown and stick-slip regime"

11:25 P. T. Phạm, C. Tsakmakis, M. Staat
«*An upper bound algorithm for limit and shakedown analysis of bounded linear kinematic hardening bodies*»

12:05 T. N. Trần, M. Staat
«An edge-based smoothed finite element method for primal dual shakedown analysis of structures under uncertainty»

12:45 Lunch in the campus restaurant Barrois

14:30 F. Pastor, P. Thoré, D. Kondo, J. Pastor
«Limit analysis and conic programming for Gurson-type spheroïd problems»

15:10 J-W Simon, D. Weichert
«A selective algorithm for solving large-scale problems in shakedown analysis»

15:50 K. V. Spiliopoulos, T.N. Patsios
«Efficient mathematical programming procedures in the elastoplastic analysis of frames»

16:30 Coffee break, Room B16

16:45 M. Chen, A. Hachemi, D. Weichert
«Shakedown and optimization analysis of periodic composites»

17:25 O. Barrera, A.R.S Ponter, ACF Cocks
«Extension of the linear matching method to softening materials»

18:05 Break

20:00 Dinner in Lille downtown

Friday, 23th October, *Great Hall*

9:30 Welcome at the Ecole Centrale de Lille

9:40 J.F. Brunel, P. Dufrénoy, E. Charkaluk
«Rolling contact fatigue: influence of the elastoplastic model on shakedown response »

10:20 Coffee break, Council Room

10:35 S. Hasbroucq, A. Oueslati, G. de Saxce
«Elastic plastic responses of a two-bar system with temperature-dependent elastic modulus under cyclic thermomechanical loadings»

11:15 G. Hassen, P. de Buhan
«Macroscopic yield strength properties of reinforced soils: from homogenization theory to a multiphase approach»

11:55 Z. Kammoun, J. Pastor, H. Smaoui
«Limit analysis of a soil reinforced by micropile group: a decomposition approach»

12:35 Lunch in the Ecole Centrale de Lille, Council Room

14:00 L. Dormieux, D. Kondo
«Micromechanical approach to the strength of porous materials with account for interface effects»

14:40 N. Antoni, Q.S. Nguyen
«Slip-shakedown analysis and the assumption of small coupling in frictional contact»

15:20 Discussion

16:00 End of the workshop

Direct Methods - Shakedown and Limit Analysis, Direct Methods 2009, Lille, Oct. 22 - 23, 2009.

Direct Methods for the Evaluation of the Ratchet Limit for Perfect Plasticity

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Abstract

If the magnitude of a cyclic loading history is raised above the shakedown limit for a perfectly plastic solid, two primary types of behaviour are possible. The body can either enter a ratchetting regime, where an increment of inelastic displacement occurs every cycle, or it can enter a reverse plasticity regime where there is zero growth of strain per cycle but, locally, a closed cycle of plastic strain occurs. Reverse plasticity occurs for thermally loaded structures when the thermal loading is dominant and also in rolling contact problems. For the evaluation of the strength of a body with cracks, there is no shakedown regime as reverse plasticity always occurs at the crack tip. The region of loading that bounds cycles for which reverse plasticity occurs is the ratchet limit, where strain growth, as well as reverse plasticity occurs. The presentation is concerned with strategies for evaluation the ratchet limit.

The steady state cyclic state of stress for a body subjected to cyclic loading may be expressed in the general form;

$$\sigma_{ij} = \lambda\hat{\sigma}_{ij} + \bar{\rho}_{ij} + \rho^r_{ij} \qquad (1)$$

where $\hat{\sigma}_{ij}$ denotes the linear elastic solution, λ a load scaling parameter, $\bar{\rho}_{ij}$ a time constant residual stress field and ρ^r_{ij} a varying component satisfying $\rho^r_{ij}(0) = \rho^r_{ij}(\Delta t)$ for a typical cycle in the steady state, $0 \le t \le \Delta t$.

Corresponding to this cycle of stress, the cycle of plastic strain ε^p_{ij} is given by,

$$\varepsilon^p_{ij} = \bar{\varepsilon}^p_{ij} + \varepsilon^{pr}_{ij} \qquad (2)$$

Where $\bar{\varepsilon}^p_{ij}$ is the distribution of (incompatible) plastic strain which gives rise to the constant residual stress field $\bar{\rho}_{ij}$ and ε^{pr}_{ij} is the history of plastic strain that give rise to the time varying residual stress field ρ^r_{ij}. By definition $\varepsilon^{pr}_{ij}(0) = 0$ and the accumulated strain per cycle is given by $\varepsilon^{pr}_{ij}(\Delta t) = \Delta\varepsilon^p_{ij}$, a compatible distribution of strain, the ratchet rate. Hence, in the reverse plasticity regime, $\Delta\varepsilon^p_{ij} = 0$ in the steady state.

The steady state is characterised by the following minimum theorem [2]. Consider the class of all plastic strain rates $\dot{\varepsilon}^c_{ij}$ so that;

$$\int_0^{\Delta t} \dot{\varepsilon}^c_{ij}\, dt = \Delta\varepsilon^c_{ij} \qquad (3)$$

and $\Delta\varepsilon^c_{ij}$ is compatible with a displacement increment Δu^c_i. For a given λ the cyclic state is characterised by the minimum of the functional [2],

$$I(\lambda, \dot{\varepsilon}^c_{ij}) = \int_V \int_0^{\Delta t} \{\sigma^c_{ij} - (\lambda\hat{\sigma}_{ij} + \rho^c_{ij})\}\dot{\varepsilon}^c_{ij}\, dt dV \qquad (4)$$

which is positive and minimised to zero by the exact cyclic solution $\dot{\varepsilon}^c_{ij} = \dot{\varepsilon}^p_{ij}$. Here σ^c_{ij} denotes the stress at yield associated with $\dot{\varepsilon}^c_{ij}$.

For values of λ in the shakedown regime, the varying component of the residual stress field $\rho^c_{ij} = 0$ and $\dot{\varepsilon}^c_{ij} = 0$. At the shakedown limit $\lambda = \lambda^s$, then $\rho^c_{ij} = 0$ but $\dot{\varepsilon}^c_{ij}$ may be considered as infinitesimally small. Hence the result $I(\lambda^s, \dot{\varepsilon}^c_{ij}) \ge 0$ yields the upper bound shakedown theorem [1]. It is important to notice that the upper bound shakedown theorem involves an infinitesimally small plastic strain rate history. Strategies for the minimisation of I, using the Linear Matching Method [1], as well as a number of programming methods are well developed, resulting in efficient upper bound methods for the shakedown limit

At the ratchet limit the strain rate history consists of two components $\dot{\varepsilon}_{ij}^{c} = \dot{\varepsilon}_{ij}^{Pc} + \dot{\varepsilon}_{ij}^{Rc}$ where $\dot{\varepsilon}_{ij}^{Pc}$ is a finite history satisfying,

$$\int_0^{\Delta t} \dot{\varepsilon}_{ij}^{Pc}\, dt = \Delta\varepsilon_{ij}^{Pc} = 0 \qquad (5)$$

and $\dot{\varepsilon}_{ij}^{Rc}$ is an infinitesimal strain rate history satisfying

$$\int_0^{\Delta t} \dot{\varepsilon}_{ij}^{Rc}\, dt = \Delta\varepsilon_{ij}^{Rc} \neq 0 \qquad (6)$$

In parallel,

$$I\left(\lambda, \dot{\varepsilon}_{ij}^{c}\right) = I\left(\lambda, \dot{\varepsilon}_{ij}^{Pc}\right) + I\left(\lambda, \dot{\varepsilon}_{ij}^{Rc}\right) \qquad (7)$$

$$I\left(\lambda, \dot{\varepsilon}_{ij}^{Pc}\right) = \int_V \int_0^{\Delta t} \{ \sigma_{ij}^{Pc} - (\lambda\hat{\sigma}_{ij} + \rho_{ij}^{Pc})\}\dot{\varepsilon}_{ij}^{Pc} dtdV \qquad (8)$$

$$I\left(\lambda, \dot{\varepsilon}_{ij}^{Rc}\right) = \int_V \int_0^{\Delta t} \{ \sigma_{ij}^{Pc} - (\lambda\hat{\sigma}_{ij} + \rho_{ij}^{Pc})\}\dot{\varepsilon}_{ij}^{Rc} dtdV \qquad (9)$$

Hence the ratchet limit is characterised by the simultaneous minimisation of $I\left(\lambda, \dot{\varepsilon}_{ij}^{Pc}\right)$ and $I\left(\lambda, \dot{\varepsilon}_{ij}^{Rc}\right)$. This has been achieved for cases where the elastic stress history is subdivided into a varying and a constant component when the two minimum problems may be solved in sequence. A particularly simple method is possible of the load history vary between two extremes [3,4,5] and may be extended to more complex histories [6]. But the primary interest is for a general method where a single load parameter λ is involved and this will be the subject of the presentation.

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On the effectiveness of numerical algorithms for the evaluation of the shakedown and limit loads

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ABSTRACT

For structures subject to a combination of loads varying within a given load domain the static and kinematic shakedown theorems, including the limit analysis theorem as a special case, furnish the safety factor against plastic collapse, loss in functionality due to excessive deformation or collapse due to low cycle fatigue.

Based on these theorems the so called *direct methods* evaluate the safety factor solving a convex optimization problem, that for real structures discretized by means of finite elements, usually require the solution of large problems. In the last decade starting from the Karmarkar algorithm the interior point method revolution [2] has completely changed the way of solving convex nonlinear optimization problems. In particular efficient primal–dual interior point methods have been developed for second order conic programming [3] to solve large problems with hundreds of thousands of variables and constraints in a reasonable computational time. Recently this method has also been applied to the reconstruction of the equilibrium path of elastoplastic structures [4].

In an alternative fashion the safety factor can be evaluated by means of the complete reconstruction of the equilibrium path, using standard strain driven strategies based on a return mapping scheme and its extension to shakedown [1]. While theoretically different, interior point methods and strain driven path–following algorithms, are very similar from a computational point of view. In this work a comparison is made to show the analogies of these methods. In particular will be shown as strain–driven like procedures can be seen as a primal optimization methods generating a converging sequence of safe states in the sense of the static (primal) theorem. This sequence, at the solution, also satisfies the requirement of the kinematical (dual) theorem.

In the paper a series of different algorithm to solve the problems and a comparisons respect to accuracy, robustness and performances are presented.

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The role of elastoplastic material models in fretting-fatigue: shakedown and stick-slip regime

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keywords: fretting, fatigue, stick, slip, shakedown, finite elements, friction

This paper presents a qualitative elastoplastic analysis of fretting and the prediction of crack initiation. We emphasize the comparison of three hardening behaviours in this context. The computational analysis is based on the estimation of the shakedown limit cycle and a fatigue prediction using a Dang Van or a Crossland criterion.

The studied configuration is the interaction of a flat pad having rounded corners in contact with a flat substrate made of Inconel In718 or Titanium Ti64 alloys respectively.

The shakedown state is analyzed using the cyclic and ratcheting strain concepts already discussed in the literature.

The paper introduces a new variable to analyse the stick-slip regime. A series of slip maps is completely analysed showing that the hardening models do not introduce a significant difference between the stick-slip or fatigue predictions.

An upper bound algorithm for limit and shakedown analysis of bounded linear kinematic hardening bodies

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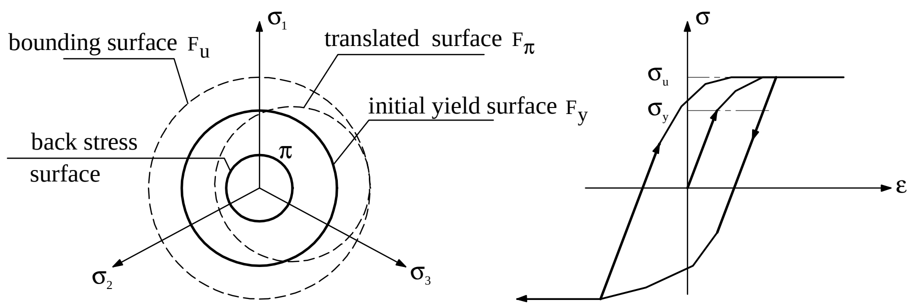
1. Introduction

FEM-based limit and shakedown analyses of industrial problems with structures with kinematic hardening materials, based on lower bound approach have been developed and implemented in the general purpose FEM codes in [1]. In [2], [3] kinematic shakedown FE-analyses have been performed for perfectly plastic materials. The upper bound theorems have been extended to bounded kinematic hardening in [4], [5]. In this work we derive an upper bound FE-algorithm for bounded linear kinematic hardening bodies. It is validated by analytical solutions for bounded kinematic hardening with the linear Melan-Prager rule and also compared to the nonlinear Amstrong-Frederick model [6], [7]. Shakedown of this nonlinearly hardening material has been treated in [8].

2. Melan-Prager bounded linear kinematic hardening model

The linear Melan-Prager kinematic hardening model can be combined with a bounding surface to a two-surface plasticity model, see Fig. 1. For realistic materials, the stress $\boldsymbol{\sigma}$ is bounded by the ultimate stress σ_u. The yield surface can translate inside bounding surface constrained by back stress π, without changing its shape and size. If σ_u is equal to σ_y, then bounding surface coincides to the initial yield surface, the two-surface model becomes a one-surface model, and consequently, kinematic hardening becomes perfect plasticity. If $\sigma_u/\sigma_y \geq 2$, the situation leads to the unbounded model.



a) Projection of surfaces on deviatoric stress plane.

b) unidimensional curve

Figure 1: A model for bounded kinematic hardening

The current surface for von Mises material is determined as:

$$F\left[\boldsymbol{\sigma}-\boldsymbol{\pi}\right] \leq \sigma_y^2 \tag{1}$$

and its corresponding dissipation function D_π^p is:

$$D_\pi^p\left(\dot{\varepsilon}_\pi^p, \dot{\varepsilon}_u^p\right) = \sigma_y \sqrt{\tfrac{2}{3}\left(\left(\dot{\varepsilon}_\pi^p + \dot{\varepsilon}_u^p\right):\left(\dot{\varepsilon}_\pi^p + \dot{\varepsilon}_u^p\right)\right)}\ . \tag{2}$$

The bounding surface for von Mises material is determined as:

$$F\left[\boldsymbol{\pi}\right] \leq \left(\sigma_u - \sigma_y\right)^2 \tag{3}$$

and its corresponding dissipation function D_u^p is:

$$D_u^p\left(\dot{\boldsymbol{\varepsilon}}_u^p\right) = \left(\sigma_u - \sigma_y\right)\sqrt{\tfrac{2}{3}\left(\dot{\boldsymbol{\varepsilon}}_u^p : \dot{\boldsymbol{\varepsilon}}_u^p\right)} \,. \tag{4}$$

The total plastic strain tensor $\boldsymbol{\varepsilon}^p$ and plastic strain rate tensor $\dot{\boldsymbol{\varepsilon}}^p$ are composed of those two components:

$$\boldsymbol{\varepsilon}^p = \boldsymbol{\varepsilon}_\pi^p + \boldsymbol{\varepsilon}_u^p \,, \qquad \dot{\boldsymbol{\varepsilon}}^p = \dot{\boldsymbol{\varepsilon}}_\pi^p + \dot{\boldsymbol{\varepsilon}}_u^p \,. \tag{5}$$

The total plastic dissipation over V defined as:

$$W_p\left(T\right) = \int_0^T dt \int_V D_u^p\left(\dot{\boldsymbol{\varepsilon}}_u^p\right) dV + \int_0^T dt \int_V D_\pi^p\left(\dot{\boldsymbol{\varepsilon}}_\pi^p, \dot{\boldsymbol{\varepsilon}}_u^p\right) dV \,. \tag{6}$$

3. An upper bound algorithm

For a FE-discretization the upper bound shakedown limit α_{blkh}^+ is obtained as the minimum

$$\alpha_{blkh}^+ = \min \sum_{k=1}^{m}\sum_{i=1}^{NG}\sqrt{\tfrac{2}{3}}\, w_i \left[\left(\sigma_u - \sigma_y\right)\sqrt{\dot{\boldsymbol{\varepsilon}}_{uik}^T \mathbf{D} \dot{\boldsymbol{\varepsilon}}_{uik} + \varepsilon_0^2} + \sigma_y \sqrt{\left(\dot{\boldsymbol{\varepsilon}}_{\pi ik} + \dot{\boldsymbol{\varepsilon}}_{uik}\right)^T \mathbf{D}\left(\dot{\boldsymbol{\varepsilon}}_{\pi ik} + \dot{\boldsymbol{\varepsilon}}_{uik}\right) + \varepsilon_0^2}\right] \quad \text{(a)}$$

$$s.t: \begin{cases} \sum_{k=1}^{m}\left(\dot{\boldsymbol{\varepsilon}}_{\pi ik} + \dot{\boldsymbol{\varepsilon}}_{uik}\right) = \mathbf{B}_i \mathbf{u} & \forall i = \overline{1, NG} \quad \text{(b)} \\ \mathbf{D}_M\left(\dot{\boldsymbol{\varepsilon}}_{\pi ik} + \dot{\boldsymbol{\varepsilon}}_{uik}\right) = \mathbf{0} & \forall i = \overline{1, NG} \quad \forall k = \overline{1, m} \quad \text{(c)} \\ \sum_{k=1}^{m}\sum_{i=1}^{NG} w_i \left(\dot{\boldsymbol{\varepsilon}}_{\pi ik} + \dot{\boldsymbol{\varepsilon}}_{uik}\right)^T \boldsymbol{\sigma}_{ik}^E = 1 & \text{(d)} \end{cases} \tag{7}$$

where $\dot{\boldsymbol{\varepsilon}}_{\pi ik}$, $\dot{\boldsymbol{\varepsilon}}_{uik}$, $\boldsymbol{\sigma}_{ik}^E$ denote respectively the vectors of back strain rate, ultimate strain rate, and fictitious elastic stress at Gaussian point i and load vertex k; $\mathbf{u}$ is the nodal displacement vector, $\mathbf{B}_i$ is deformation matrix; $m = 2^n$, n is the number of varying loads; NG is the total number of Gaussian points of the whole structure with integration weight w_i at Gaussian point i, ε_0 is a small parameter of regularization. $\mathbf{D}$ is a diagonal square matrix. In a three-dimensional problem it is the 6×6 matrix:

$$\mathbf{D} = Diag\begin{bmatrix} 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}. \tag{8}$$

For the sake of simplicity, some new notations in (9) will be used

$$\begin{cases} k_v = \dfrac{\sigma_y}{\sqrt{3}} & \text{: crirtical value, von Mises yield criterion} \\ \dot{\mathbf{e}}_{\pi ik} = w_i \mathbf{D}^{1/2} \dot{\boldsymbol{\varepsilon}}_{\pi ik}, \ \dot{\mathbf{e}}_{uik} = w_i \mathbf{D}^{1/2} \dot{\boldsymbol{\varepsilon}}_{uik} & \text{: new strain rate vectors} \\ \mathbf{t}_{ik} = \mathbf{D}^{-1/2} \boldsymbol{\sigma}_{ik}^E & \text{: new fictitious elastic stress field} \\ \hat{\mathbf{B}}_i = w_i \mathbf{D}^{1/2} \mathbf{B}_i & \text{: new deformation matrix} \end{cases} \tag{9}$$

Then problem (7) could be rewritten as:

$$\alpha_{blkh}^+ = \sqrt{2} k_v \min \sum_{k=1}^{m}\sum_{i=1}^{NG}\left[\frac{\left(\sigma_u - \sigma_y\right)}{\sigma_y}\sqrt{\dot{\mathbf{e}}_{uik}^T \dot{\mathbf{e}}_{uik} + \varepsilon^2} + \sqrt{\left(\dot{\mathbf{e}}_{\pi ik} + \dot{\mathbf{e}}_{uik}\right)^T \left(\dot{\mathbf{e}}_{\pi ik} + \dot{\mathbf{e}}_{uik}\right) + \varepsilon^2}\right] \quad \text{(a)}$$

$$s.t: \begin{cases} \sum_{k=1}^{m}\left(\dot{\mathbf{e}}_{\pi ik} + \dot{\mathbf{e}}_{uik}\right) = \hat{\mathbf{B}}_i \mathbf{u} \quad \forall i = \overline{1, NG} & \text{(b)} \\ \frac{1}{3}\mathbf{D}_M\left(\dot{\mathbf{e}}_{\pi ik} + \dot{\mathbf{e}}_{uik}\right) = \mathbf{0} \quad \forall i = \overline{1, NG} \quad \forall k = \overline{1, m} & \text{(c)} \\ \sum_{k=1}^{m}\sum_{i=1}^{NG}\left(\dot{\mathbf{e}}_{\pi ik} + \dot{\mathbf{e}}_{uik}\right)^T \mathbf{t}_{ik} = 1 & \text{(d)} \end{cases} \tag{10}$$

Let α_{pp}^{+} and α_{blkh}^{+} denote respectively the upper bound shakedown limit load in perfect plasticity, and in bounded linear kinematic hardening with the same yield stress σ_y, then:

$$\alpha_{pp}^{+} \le \alpha_{blkh}^{+} \le \frac{\sigma_u}{\sigma_y}\alpha_{pp}^{+} . \tag{11}$$

The first equality occurs when $\dot{\bar{u}}_{ik} = 0$, (existing yield surface is strictly below bounding surface), and the second equality occurs when $\dot{\bar{\pi}}_{ik} = 0$, (existing yield surface is fixed on bounding surface). In all other cases strict inequalities $\alpha_{pp}^{+} < \alpha_{blkh}^{+} < \frac{\sigma_u}{\sigma_y}\alpha_{pp}^{+}$ occur when $\dot{\bar{\pi}}_{ik} \neq 0$ and $\dot{\bar{u}}_{ik} \neq 0$, (existing yield surface moves on bounding surface).

4. Validations

The test has been done for the structure in Fig. 2a, subjected to combined tension and torsion loading:

$$\begin{cases} \text{Tension: } N\text{: dead load} \\ \text{Torsion: } M \in \left[-M_{\max}, M_{\max}\right] \end{cases} \tag{12}$$

The material: $\sigma_y = 485$ MPa, $\sigma_u / \sigma_y = 1.3$. The geometry: $R_a = 12.7$ cm, $R_i = 11.17$ cm.
The interaction diagram of the load factors of tension vs torsion in the Fig. 2b is normalized by limit loads of perfect plasticity material.

Analytical solutions

With the load domain in (12), the shakedown condition for the bounded linear Melan-Prager model is [7]:

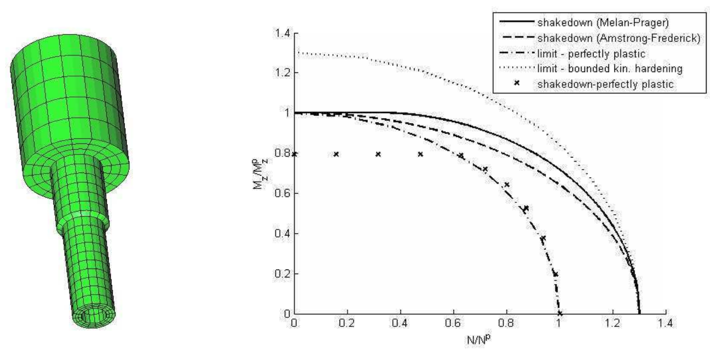
$$\tau_{mp} = \begin{cases} \frac{1}{\sqrt{3}}\sigma_y & \text{for } 0 \le \sigma_N \le \left(\sigma_u - \sigma_y\right) \\ \frac{1}{\sqrt{3}}\sqrt{\sigma_y^2 - \left(\sigma_N + \sigma_y - \sigma_u\right)^2} & \text{for } \left(\sigma_u - \sigma_y\right) < \sigma_N \le \sigma_u \end{cases} \tag{13}$$

and for the nonlinearly kinematic hardening Amstrong-Frederick model is [7]:

$$\tau_{af} = \frac{1}{\sqrt{3}}\frac{\sigma_y}{\sigma_u}\sqrt{\sigma_u^2 - \sigma_N^2} . \tag{14}$$

FE analysis

Using 20-node volume elements, if $\sigma_u / \sigma_y = 1$ the results exactly the same of perfectly plastic are obtained. It is expected that the results close to the curves plotted for $\sigma_u / \sigma_y = 1.3$ in Fig. 2 for the Melan-Prager and Amstrong-Frederick models are found.



(a) FEM mesh (b) interaction diagram of plastic and shakedown factors

Figure 2: FEM mesh and interaction diagram of plastic and shakedown factors

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An edge-based smoothed finite element method for primal-dual shakedown analysis of structures under uncertainty

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Abstract

The plastic collapse limit and the shakedown limit which define the load-carrying capacity of structures are important in assessing the structural integrity. Due to the high expenses of experimental setups and the time consuming full elastic-plastic cyclic loading analysis, the determination of these limits by means of numerically direct plasticity methods has been of great interest to many designers. Moreover, a certain evaluation of structural performance can be conducted only if the uncertainty of the actual load-carrying capacity of the structure is taken into consideration since all resistance and loading variables are random in nature. To ensure the safety of the structures to be designed, two approaches are normally used. (1) The classical approach fixes the values of the safety factors and chooses the values of the design variables to satisfy the safety conditions. All the variables involved are then assumed to be deterministic and fixed to particular quantiles, i.e. mean value or characteristic values. (2) The probability-based approach deals directly with realistic random variables to find the global probability of failure as the basic design criterion. Obviously, the later problem is more difficult since the evaluation of the probability of failure is not an easy task [1], [2].

Effective method for stress analysis is essential and of fundamental importance for structural reliability analysis. Very recently, Liu et al. [3] proposed an edge-based smoothed finite element method (ES-FEM) for static, free and forced vibration analyses of solid 2D mechanics problems using triangular elements (T3). Intensive numerical results have demonstrated that ES-FEM possesses the following excellent properties: (1) ES-FEM-T3 is much more accurate than the FEM using linear triangular elements (FEM-T3) and often found even more accurate than those of the FEM using quadrilateral elements (FEM-Q4) with the same sets of nodes; (2) there are no spurious non-zeros energy modes found and hence the method is also temporally stable and works well for vibration analysis and (3) no penalty parameter is used and the computational efficiency is much better than the FEM using the same sets of elements. The ES-FEM was then developed for static and eigenvalue analysis of two-dimensional piezoelectric structures [4].

This paper aims at presenting a new algorithm for probabilistic limit and shakedown analysis of structures with the help of the ES-FEM. The algorithm includes a deterministic limit and shakedown analysis for each iteration step which is based on the primal-dual approach. The loading and material strength are to be considered as random variables. The limit state function separating the safe and failure regions is defined directly as the difference between the obtained limit load factor and the current load factor. A Sequential Quadratic Programming (SQP) is performed for finding the most probable failure point, the so-called design point. Sensitivity analyses are obtained numerically from a mathematical problem and the probability of failure is calculated by the FORM. Using constant smoothing function which leads to local constant smoothing domains constructed on edges of elements, only one Gaussian point is required for each

smoothing domain ensuring that the total number of variables in the resulting optimization problem is kept to a minimum compared with standard finite element formulation. Moreover, this results in a true lower bound coupling with a true upper bound obtained by displacement-based finite element method since the yield condition is fulfilled at all points in the problem domain. This direct approach reduces considerably the needs for uncertain technological input data, computing costs and the numerical error [5, 6].

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Limit analysis and conic programming for Gurson-type spheroïd problems

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In the famous paper [4], Gurson proposed an upper bound limit analysis approach of a hollow sphere with a von Mises solid matrix. The computation has been performed under uniform boundary strain rate conditions and led to a macroscopic yield function of the "Porous von Mises"-type materials.

Several extensions have been further proposed in the literature, the most probably important developments being those accounting for voids shape effects [2], [1], [5]. More recent extensions of the Gurson model deal with the plastic compressibility of the matrix with Drucker-Prager criterion, as it is the case for polymers or cohesive geomaterials [3]. For validation purpose of these recent models, and for reference solutions in the compressive matrix case, there is a need of numerical solutions, which is the purpose of the present communication.

First, in the case of a Drucker-Prager matrix and for spherical cavities we analyze the possibilities of combining the exact spherical solution for the hollow sphere model with a homogeneous strain rate field. Using the kinematical approach under axisymmetry assumption, the resulting problems are analytical conic programming ones, solved with the specific code MOSEK. The final results show the impossibility, using such two velocity fields, to obtain a good *estimate* of the macroscopic criterion without relaxing the local plastic admissibility as made in [3].

To obtain pertinent rigorous bounds to the exact solutions in terms of limit analysis, we have improved the kinematical and static 3D-FEM codes of [7]. The kinematical problem is formulated in such a way that either the Drucker-Prager and von Mises criteria can be indifferently solved. In both codes, using specific changes of variables, the resulting formulations have allowed to significantly refine the discretization and to obtain better bounds to the macroscopic criteria for spherical cavities. Finally the discretization of the hollow sphere was modified to take into account the case of oblate cavities confocal with the spheroid boundary.

Extended comparisons with the previously mentioned works (then for von Mises matrices) will be presented, in both cases of Hill-Mandel homogeneous conditions and for oblate cavities. Then, we give a detailed comparison with the "Porous Drucker-Prager" estimation of [3], and first results for oblate cavities where the bounds are acceptable for friction angles lower than twenty degrees, but too distant above this angle value for the moment.

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A selective algorithm for solving large-scale problems in shakedown analysis

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This work is focused on the calculation of shakedown load-factors of structures subjected to variable loads by using the lower-bound theorem of shakedown analysis. The resultant optimization problem is solved by an algorithm based on the Interior Point method which has already proven its capability to solve industrially important problems. In general, the solution of such large-scale problems leads to a large number of unknowns and constraints and thus turns out to be very computationally intensive in many cases. In order to reduce calculation time without unacceptable loss of accuracy a new selective algorithm is introduced that only takes into account those constraints that belong to so-called 'active' elements.

EFFICIENT MATHEMATICAL PROGRAMMING PROCEDURES IN THE ELASTOPLASTIC ANALYSIS OF FRAMES

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Elastoplastic analysis is an important procedure to determine the capacity of a structure beyond its elastic limits. In the course of this analysis, the external loads are continuously applied with more and more structural components yielding. A series of elastic analyses are therefore generated by modifying the mathematical model of the structure to account for reduced resistance of yielding components. The procedure consists of the superposition of these analyses and stops when the structure cannot carry any further load and becomes unstable, or until a predetermined load limit is reached. Thus a good estimate of the strength of the structure as well as of its ductility can be made.

The force method of analysis is well suited to formulate a plasticity problem since the main conditions, i.e. equilibrium conditions, are satisfied in an exact manner throughout the whole structure. The difficulty of the method is its automation. This may be provided by the graph representation of a frame. In this representation, a member and a node of the frame are a member and a node of a directed graph, respectively. The foundation node may be considered as an extra node and each foundation node is connected to the ground node with an extra member; then there is a unique number of independent closed loops, called the Betti number. The automatic way to unveil these loops [1] provides a set of hyperstatic forces to be utilised within the force method.

A novel numerical approach, which uses the algorithm [1] together with a mathematical programming algorithm, will be discussed in the present work. The elastoplastic problem, as described in the first paragraph, is formulated as a, plastic hinge based, incremental one, that requires the solution of a parametric convex quadratic programming (PQP) problem between two successive plastic hinges. A fictitious load factor is used to convert the PQP problem to a QP one. The solution of the QP problem by an effective algorithm [2], establishes a feasible direction on which the true solution lies. The real solution is then found, simply on the demand of the formation of a new plastic hinge that is closest to open. Possible plastic unstressing is automatically accounted for [3]. Two different numerical procedures for either pure bending or for bending and axial force interaction, based on the above described concept, will be presented. Examples of application under monotonic or variable loading will also be included. Results show that the procedures are stable and computationally efficient as they require much less time than the alternative procedures which are based on the direct stiffness method.

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Shakedown and optimization analysis of periodic composites

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Abstract:

Based on homogenization theory and large scale nonlinear programming, this paper investigates the elastic and plastic properties of periodic composites subjected to variable loading. After evaluation of homogenized elastic and plastic material properties of composites, shakedown and limit analysis are performed in macroscopic scope for the further plastic design. The optimal design variables, such as fiber distribution and various volume fractions are also investigated.

EXTENSION OF THE LINEAR MATCHING METHOD TO SOFTENING MATERIALS

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ABSTRACT:

An extension of the Linear Matching Method (LMM) is described which takes into account material softening. . It is well known that localized strain softening behavior can result in the premature failure of structural components. There are a number of circumstances where softening is relevant, for example: local buckling of beams in portal frames; local buckling in sandwich shell structures; and degradation in strength of composite structures due to internal cracking and fiber failure. For beam type elements the local constitutive instability is evidenced by a decrease of the bending moment with increasing flexural rotation. Different methods have been proposed in the literature to evaluate the response of structural systems that exhibit local softening. The most common approach consists of a step-by-step analysis of the behaviour under increasing load. Even though this type of approach can provide a detailed description of the structural response, it is computationally demanding. Direct or bounding methods are more ideally suited to the initial stages of design. Cocchetti and Maier [1] have consolidated and summarised the application of mathematical programming methods to the limit analysis of portal frames. These methods have been extended to the behavior of elastic-softening plastic portal frames by Ferris and Tin-Loi [2] and Tangaramvong and Tin-Loi [3,4], who emphasize the importance of Mathematical Programming with Equilibrium Constraints (MPEC) methods. In this context, the main objective of the present work is to investigate the extension of the Linear Matching Method to evaluate the maximum load that an elastic-plastic frame structure can withstand when material or element softening is present. Linear Matching Methods are a class of programming methods where, at each iteration, equilibrium and compatibility are satisfied and convergence is imposed by ensuring material consistency. Convergence proofs have been derived for classical limit analysis by Ponter et al [5] and shakedown by Ponter and Engelhardt [6].

In this paper a three step LMM procedure is described which systematically evaluates the structural response for different level of softening. Consider a structure composed of an elastic-plastic material that exhibits softening The constitutive behavior is assumed to be holonomic, neglecting any ‘‘local unloading’’ that can occur. We consider the moment-rotation relationship indicated in Fig.1, where three different regions can be distinguished: elastic region (*e*), plateau region (δ), and softening region (*s*). The structure is subjected to a set of proportional loads λF_i and the objective is to find the nodal displacements Δ_i^c and rotations $\Phi_j^{\ c}$ that will yield the largest value of the load factor λ, indicated by λ^{MAX} .

The process of evaluating the maximum load consists of the following three major steps:

1) The LMM is employed to determine the maximum load at which $\Phi \le \Phi_c^I$ throughout the structure, where Φ_c^I is the curvature at the start of the softening region as illustrated in fig 1 .
2) The range of values of the softening slope *P* for which the load can be increased beyond that determined in (1) is identified.
3) For values of *P* which satisfy the criterion established in (2) the maximum load , λ^{MAX} ,that the structure can withstand is determined through a two stage iterative procedure based on the LMM.

In order to illustrate the method, we consider the response of the single story portal frame of Fig 2, which is subjected to an horizontal load *H* and a vertical load *V*=α*H*. We present results for L=10m and α =0.25, with the horizontal load *H*

identified with λ L/*Mc*. We consider the situation where the initial rotational stiffness *R*=45 kNm and $\phi_c = 0.033$, so that the plastic moment $M_c = 1.48$ kNm. We assume the extent of the plateau region $\delta = \phi_c^I - \phi_c = 0.01$. The first step provides the value of the maximum load $\lambda_{\delta=0.01}^{MAX}$ at which $\Phi \le \Phi_c^I$ (Fig 1). The associated maximum load is $\lambda_{\delta=0.01}^{MAX} = 0.576$. The second step is employed to find the value of the critical slope *Pcrit*, below which the load can be increased beyond $\lambda_{\delta=0.01}^{MAX} = 0.576$. We find *Pcrit* is equal to 22 kNm. Results for the third step of the procedure are shown in Fig 3. In less than ten iterations the static and kinematic bounds to the maximum load multiplier coincide, giving the exact solution for λ^{MAX}, which is a function of the slope *P*, as illustrated in Fig 3.. When *P*=0 the constitutive model reduces to perfectly plastic material behaviour, so that the maximum load is the limit load for the structure (λ_L). It can be seen in Fig 3 that λ^{MAX} decreases monotonically from the limit load multiplier $\lambda_L = 0.594$ to $\lambda_{\delta}^{MAX} = 0.576$ as *P* is increased from 0 to 22 kNm.

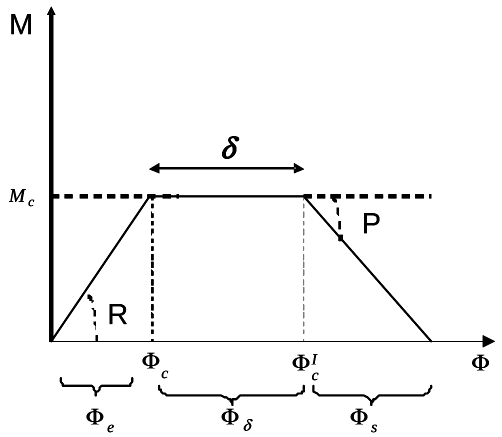

Figure.1

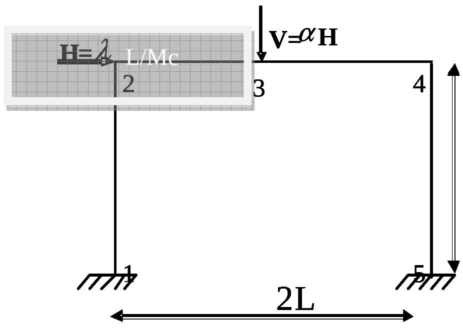

Figure.2

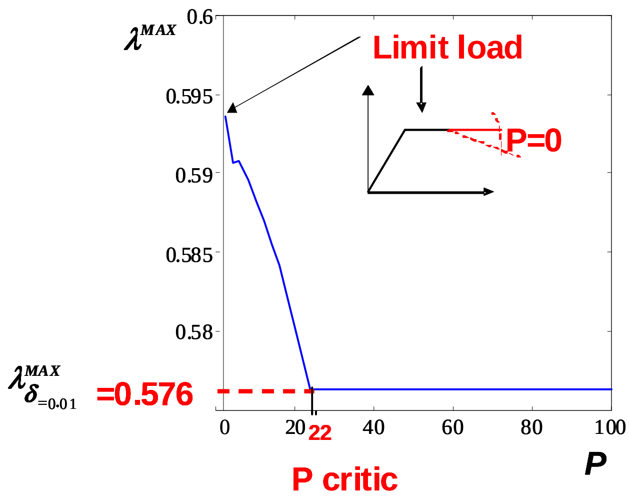

Figure.3

The method has been applied to frame structures with increasing numbers of degrees of freedom . It has been noted that stable solutions are obtained for high and low levels of softening, but numerical instabilities in the procedure can occur for intermediate degrees of softening, in the vicinity of *Pcrit*

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Rolling contact fatigue: influence of the elastoplastic model on shakedown response.

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With the increase of train speeds and axle loads, rolling contact fatigue of railway wheels has become an important issue with respect to failure. Three types of fatigues in wheels are accounted for: surface initiated fatigue, subsurface initiated fatigue and fatigue initiated at deep material defects. Even if simplified models, like the shakedown map, may be used to analyze and predict the surface fatigue, such approaches are not sufficiently precise to compare different steel grades. Two steels usually used for the manufacturing of wheels have been studied: R9T, 50CrMo4.
The aim of this work is to develop an approach which is able to compare the different steel grades according to practical conditions. The main stages are the identification of the material behavior, the determination of the stress-strain fields and the application of a fatigue criterion.

The identification of material behavior has been done from cyclic mechanical tests. Plastic models with linear and non linear kinematic hardening laws have been identified for the numerical model.
Stress and strain responses have been calculated using an eulerian description of the wheel-rail contact in order to limit the size of the finite element model. Computations have been performed with Abaqus software. Several cases of loading have been explored by varying the vertical load and the sliding velocity. The stress and strain fields are determined in stationary regime. Numerical results give several cases of mechanical behavior: elasticity, elastic shakedown, plastic shakedown and ratcheting. Simulations are realized and compared with the shakedown map results.
As a first step, the Crossland multiaxial fatigue criterion has been used for the high cycle fatigue analysis in the case of elastic shakedown.

The different steel grades have been compared in terms of mechanical behavior for several levels of vertical loads and friction coefficients. Results show that the threshold of elastic and plastic shakedown differs depending on the steel grades and consequently the risk of damage can be affected.

Finally, the results are analyzed in terms of level of in-phase and out-of-phase stress loading, hydrostatic pressure and shear stresses showing the limits of the Crossland criterion and the difficulties to identify an accurate fatigue criterion.

This methodology allows a classification of the material grades face the risk on rolling contact fatigue.

Elastic plastic responses of a two-bar system with temperature-dependent elastic modulus under cyclic thermomechanical loadings

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This work is concerned with the inelastic responses of a two-bar system with temperature-dependent elastic coefficients under cyclic thermomechanical loadings. It is found that the structure evolves toward a periodic limiting state as for classic elastoplasticity. In order to understand how Melan-Koiter method works for such materials, the evolution of the structure's response until the stabilization of the plastic strain (shakedown) or the asymptotic dissipative behaviour (alternating plasticity or ratchetting) is analytically addressed and the Miller's diagram is then constructed. In passing, we show that shakedown results results present a counter example to the Halphen's shakedown conjecture for materials with temperature-dependent elastic properties. Finally, numerical results performed by an incremental finite element procedure are presented and compared to analytical ones.

MACOSCOPIC YIELD STRENGTH PROPERTIES OF REINFORCED SOILS: FROM HOMOGENIZATION THEORY TO A MULTIPHASE APPROACH

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This contribution addresses the problem of evaluating the overall yield strength properties of soils reinforced by linear inclusions from those of their individual components, as well as from the reinforcement volume fraction and orientation. This question may be dealt with by resorting to the *yield design* (or limit analysis) *homogenization theory* ([1], [2]), according to which the macroscopic strength criterion of the composite reinforced soil can be determined from the solution to a yield design boundary value problem defined over the reinforced soil’s representative volume. In the commonly encountered situation when the soil is reinforced by continuous linear inclusions of small cross section, but made of a highly resistant material (metal, concrete), the macroscopic strength condition can be given a fully analytical formulation ([3]), which clearly reveals the strength anisotropy of the homogenized reinforced soil, due to the reinforcement preferential orientations.

The validity of such a yield design homogenization procedure is assessed on the illustrative problem of the plane strain compressive resistance of a transversely reinforced block. A comparison is presented between the homogenization-based prediction and numerical simulations performed on the composite specimen where the reinforcements are regarded as individual elements embedded in the soil. It is shown that the numerical predictions do converge to that derived from the homogenization procedure as a *scale factor,* defined as the ratio between the spacing between two adjacent reinforcing inclusions and the overall size of the reinforced specimen, tends to zero. From an engineering standpoint, the practical applicability of such a mathematical convergence result requires that the scale factor be sufficiently small, which is obviously the case for industrial composite materials, but remains highly questionable for reinforced soil structures, since in most cases such a scale factor may be of the order of 0.1.

Conceived as an extended or generalized homogenization procedure, a *multiphase model* is then advocated for ascertaining the overall yield strength of reinforced soils, taking into account the previously mentioned *“scale effect”* in an explicit way. According to this model, the reinforced soil is perceived as the superposition of two continuous media, namely the matrix phase representing the soil on the one hand, the reinforcement phase, which represents the array of reinforcing inclusions, on the other hand. While the strength properties of each phase are described by means of a failure condition, a specific strength condition can be assigned to the interaction forces between both phases. Provided that such an interaction strength parameter be properly identified, the above mentioned scale effect can be fully captured by the two-phase model, thus recovering the results of the numerical simulation.

The results presented in this contribution are the "yield design/limit analysis" equivalent of those already established in [5] in the context of a linear elastic behaviour.

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LIMIT ANALYSIS OF A SOIL REINFORCED BY MICROPILE GROUP:
A DECOMPOSITION PPROACH

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Astract.

The behavior of soils reinforced by micropile networks, first developed in the fifties [1], is still not fully understood due to the lack of accurate modelling capabilities.

Particularly, the complex geometry of large soil-micropile systems makes accurate calculation of the bearing capacity of the reinforced soil a computational challenge. This complexity, arising from the high aspect ratio of micropiles and the large number of micropiles typically composing a network, requires highly detailed and finely discretized models to achieve reasonable accuracy using direct numerical methods. Such models lead to large scale numerical optimization problems that are hardly tractable using a personal computer.

Homogenization techniques [2] constitute an attractive alternative for efficiently modelling the behavior of the soil-micropile medium, however, inevitable homogenization errors remain significant near the interface with the natural soil.

In a recent work [3], a decomposition method has been proposed as a strategy for systematically solving very large kinematic limit analysis problems with limited computing resources. It consists of splitting the original problem into limit analysis subproblems that are smaller in size. In [4] the static implementation of the decomposition was applied to a compressed bar and the classical vertical cut problems.

The proposed paper reports enhancements made to the original decomposition method, allowing the method to solve the classical punch problem presented as a test problem, and then describes the application of the decomposition method to determine rigorous kinematic and static bounds to the bearing capacity of a soil reinforced by a micropile network according to a 2D plane strain model.

The punch problem, a representation of Prandtl's classical problem with finite domain, is considered here for i/ being a simple, limit case of a soil reinforced by mircopiles, for which the solution is known a priori, that is the case with no reinforcement, ii/ exhibiting a feature that has not been tested so far in decomposition, that is the absence of a loaded zone in some subproblems.

The mixed kinematic variant of the decomposition, detailed and illustrated in [3-4] on the compressed bar and the vertical cut problems, is applied to compute upper bounds for Prandtl's punch problem and the micropile reinforced soil problem.

Then, the static adaptation of the decomposition is presented and applied for the first time on a problem exhibiting unloaded subdomains, embodied here by the punch problem. The issue pertaining to subdomains without loaded zones is that no change in the stress field within the

subdomain follows directly from variations in the global load parameter while the interface stresses are kept constant. As a result, this load parameter cannot be used as a driver of convergence of the stress field in all subdomains.
In order to force the stresses to evolve in an unloaded subdomain, the corresponding subproblem is posed as that of minimizing the cohesion of the Tresca-Coulomb (or Drucker-Prager) soil considered as a variable, while maintaining the stresses constant at the interfaces. This creates a buffer with respect to the failure criterion that allows subsequent scaling of the stress field in the unloaded subdomain, consistently with the changing loading acting on remote zones.

The soil, bounded below by a substrate at depth H, is to be reinforced by a group of four micropiles of length L with the purpose of supporting a load F. Due to symmetry, only half the domain is modeled. The numerical optimization problems arising from the finite element limit analysis (sub)problems involved in this work are all solved using the conic programming code MOSEK.

With the direct solution approach, for meshes of more than 25,000 triangles, the optimization code MOSEK fails to give optimal solutions for the static problem. Beyond this size, the problem can only be solved by the decomposition approach.
With the mixed kinematic approach, the largest mesh that can be treated by direct solution counts 10080 elements. For this mesh, the kinematic bound is 1134.6 and the static bound is 35.5. With a partition into four subdomains the solution becomes possible by decomposition with a 40320 finite element mesh . The gap between bounds is narrowed down substantially, with the static bound increased to 36.4 and the kinematic bound lowered down to 41.08. These bounds can be improved using the same computational resources by further refining the partition.

With this degree of discretization, visualization of the stress and the velocity fields at the limit state clearly reveals the behavior pattern of the soil-micropile system which is known to vary from a footing like to a pile like pattern depending on the geometric characteristics of the problem.

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Micromechanical approach to the strength of porous materials with account for interface effects

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Abstract

In the last decade, size-dependent effects in nanomaterials including materials containing nano-voids have focused the attention of many researchers. Early works have tried to model the transition zone between the nano-inclusion and the surrounding matrix as a thin but still three-dimensional layer [7],[5]. An alternative approach consists in adopting an interface description which is two-dimensional in nature. Progress has been gained in the understanding of inclusion size effects on the effective elastic properties. Classical homogenization schemes [1] as well as first order bounds in the theory of elastic heterogeneous media have been extended in order to incorporate interface and interface stresses (see e.g. [2],[6],[4]).

In contrast, it seems that few attention has been paid so far to the question of the effective strength of nanomaterials with account for interface effects. In the context of the ductile failure of porous materials, the Gurson model [3] is well known to provide a efficient approach of the strength reduction due to the porosity. The purpose of the present communication is to extend this model in order to capture the influence of interface stresses.

To begin with, in view of subsequent extensions, the basic features of the classical Gurson approach for ductile porous media are recalled. Then, they are extended to incorporate the surface/interface stresses effect at the nano-scale. For capillary forces, the yield surface is shown to be obtained by a mere translation of Gurson one. For interface stresses obeying a von Mises criterion, the parametric equations of the yield surface are derived. The magnitude of the interface effect is proved to be controlled by a non dimensional parameter depending on the voids characteristic size.

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Slip-shakedown analysis and the assumption of small coupling in frictional contact

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ABSTRACT

In the frictional contact of solids under cyclic loads, the shakedown behaviour of the relative displacement is of interest in the same spirit as the plastic deformation in plasticity. Cumulative slips may lead to the failure due to large relative displacements of the components of an assembly while cyclic slips are often undesired because of wear and fretting fatigue problems. Under Coulomb friction, it is well known that Melan and Koiter theorems are generally not available, except in certain particular cases. In this discussion, the particular case of *small coupling* between the contact pressures and the slip-displacements is considered. This assumption means that the tangent displacements have small or no influence on the contact pressures which can be then computed from the elastic response as in the *uncoupling* case. The pressure is thus a given time-dependent function and the Coulomb criterion is reduced to a Mises-like standard law of friction. It is shown here that Melan and Koiter theorems can be applied again as in standard plasticity. The dependence of the yield limit on the loading amplitude is however not classical and the extension of the static and kinematic approaches is discussed to obtain the critical shakedown load or the limit load. The validity of the assumption of small coupling is also explored by numerical simulation in an example of car conrod.