Limit States of Materials and Structures
Direct Methods

DIRECT METHODS

Shakedown and Limit Analysis

Workshop held at:

Institut für Allgemeine Mechanik
RWTH Aachen
on 08. - 09. Nov 2007

1 Summary and Conclusion

The workshop on ***Direct Methods - Shakedown and Limit Analysis*** was held on Nov. 08. - 09.2007 at Rheinisch-Westfälische Technische Hochschule (RWTH) Aachen, Germany on the initiative of Dieter Weichert (Aachen) and Alan Ponter (Leicester).

With 30 international participants and 17 presentations the workshop has to be referred to as a success. The presentations and the following discussions felt to be extremely fruitful for both the speakers and the members of the audience.

After some words of welcome by the organizers, D. Weichert gave an overview about the *Achievements and challenges in Shakedown Analysis* given by Prof. D. Weichert. Then the first talk with the title *Direct Methods for the Life Assessment of structures at high temperatures and other problems* was given by A. Ponter.

The following four presentations were concerned with the application of Shakedown and Limit Analysis to special materials, namely *poroplastic, porous frictional and periodic media as well as composites*. Lectures were held by G. Cocchetti, J. Pastor in collaboration with P. Thoré, S. Bourgeois and J.-H. You, respectively.

The next group of presentations dealed with the aspects of *Stochastic Uncertainty and Reliability Analysis*. These were given by K. Marti, S. Zier and M. Staat in collaboration with T. N. Trần.

The Thursday session was closed by G. de Sacxé's lecture *Shakedown Analysis of problems with non-associated constitutive laws and with elastic coefficients depending on the temperature*.

The presentations which were given on Friday reflected the variety of issues that can be examined in the framework of Shakedown and Limit Analysis.

The first talk was given by O. Barrera who presented her work on the application of the *Linear Matching Method on Limit Analysis of orthotropic laminates*.

The following two papers dealed with the evolution of damage considered from different point of views. While E. Charkaluk used a new *Plasticity-Damage Approach in Fatigue* which can be adopted in the framework of Direct Methods, G. Geißler followed the Step-by-step Methods for the implementation and applicatian of a *Cohesive Zone Model within the Finite Element Framework*.

In the broadest sense the next two presentations can be assigned to the subject of friction. The *Tribology and mechanical systems functionalities* in the scope of Direct Methods was introduced by G. Inglebert. M. Ciavarella then showed an extension of *Melan's theorem for the study of frictional systems*.

The *Limit Equilibrium Analysis of framed structures* with help of *Force Method based procedures* was presented by K.V. Spiliopoulos.

The session was completed by A. Hachemi's talk which was focused on the *application of Direct Methods to large-scale problems*.

The last but definitely not least topic of the agenda was a ***discussion about the possibilities for common European projects and proposals***. During this discussion the group of participants agreed upon launching a network which is intended to federate the community of researchers in the field of Direct Mehods. The suggested title for this network is **Limit States of Materials and Structures - Direct Methods**.

The network shall be open not only to the participants of the workshop but to every researcher who is interested in attending. The initiation will be coordinated by Aachen. Nevertheless the network is planned to be leaded in a democratic way without any kind of governing council although there might be the necessity of naming some spokespersons of the group. This will have to be a point of discussion in a later stage of the initiation process.

The aim of this network is on the one hand building up new connections as well as improving already existing ones and on the other hand the implementation of international projects and proposals which might possibly be funded by the *Seventh Framework Programme of the European Community for research, technological development and demonstration activities (2007-2013)*. The possibilities of this support will be checked independently by the participants taking into account the different national regulations.

An important tool for improving connectivity will be a special website which will be constructed by H. Smaoui in collaboration with D.K. Vu and with the help of the Aachen-group.

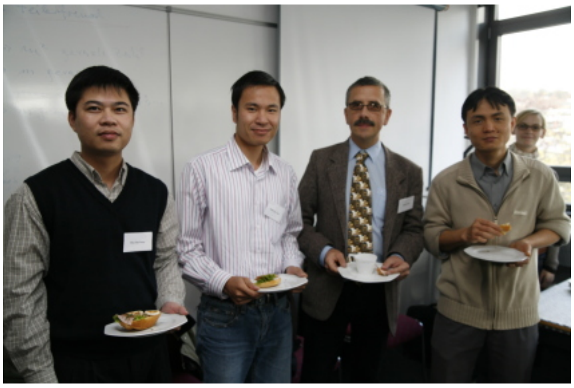
Because of the success of the workshop the participants decided to repeat this kind of meetings in regular intervals of two years at different locations.

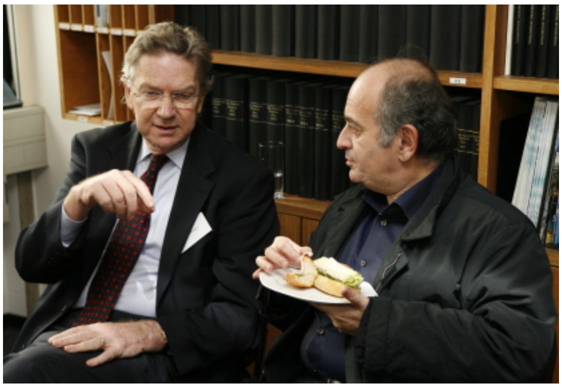
2 Impressions

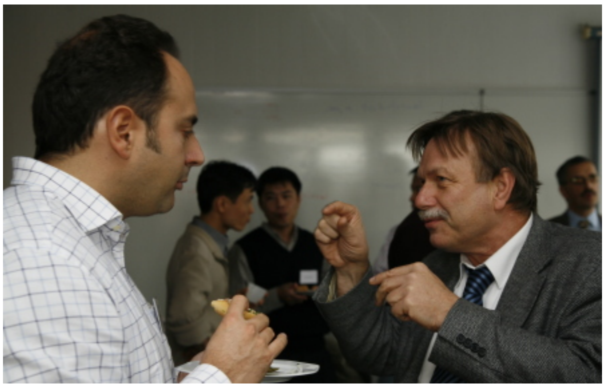
Although, of course, the main focus of the workshop lied upon the scientific effort there are other aspects worth to be mentioned.

One interesting aspect was the multinationality of the workshop's attendees. There were representatives of research groups from not less than seven countries: Belgium, France, Germany, Greece, Italy, Tunisia and UK. The number of nationalities is even higher if each participant is considered individually.

It was particularly pleasant to see the great number of young scientists who attended the meeting and contributd by their interest and freshness of mind to the friendly and optimistic atmosphere which was besides the high quality of the presentations one essential key to the success of the workshop.





3 Agenda

AGENDA Thursday 08.11.2007

9:00 - 10:00 **D. Weichert, A. Ponter:** *Reception and Introduction*

D. Weichert: Introducing overview: Achievements and challenges in Shakedown Analysis

A. Ponter: Direct Methods for the Life Assessment of structures at high temperatures and other problems

10:00 - 10:45 **G. Cocchetti:** Generalized Limit Analysis for poroplastic media

coffee break

11:00 - 11:45 **J. Pastor, P. Thoré:** Limit Analysis and optimization: yield function of porous frictional materials and Decomposition Methods

11:45 - 12:30 **S. Bourgeois:** Limit Analysis and Shakedown of periodic media. Case of plates and beames

lunch break

14:00 - 14:45 **J.-H. You:** Application of Shakedown Analysis to fiber-reinforced metal matrix composites and a composite component

14:45 - 15:30 **K. Marti:** Limit Load and Shakedown Analysis under stochastic uncertainty

coffee break

15:45 - 16:30 **S. Zier:** Limit Load Analysis of plane frames under stochastic uncertainties

16:30 - 17:15 **M. Staat, T. N. Trần:** Reliability Analysis of inelastic shell structures under variable loads

17:15 - 18:00 **G. de Saxcé:** Shakedown Analysis of problems with non-associated constitutive laws and with elastic coefficients depending on the temperature

AGENDA Friday 09.11.2007

9:00 - 9:45	**O. Barrera:** Limit Analysis of orthotropic laminates by the Linear Matching Method
9:45 - 10:30	**E. Charkaluk:** Fatigue and Shakedown concepts
coffee break	
10:45 - 11:30	**G. Geißler:** Cohesive zone model within the Finite Element Framework
11:30 - 12:15	**G. Inglebert:** Tribology and mechanical systems functionalities
12:15 - 13:00	**M. Ciavarella:** Extended Melan's theorem for the study of frictional systems
lunch break	
14:30 - 15:15	**K. V. Spiliopoulos:** Force Method based procedures in the Limit Analysis of 2D & 3D framed structures
15:15 - 16:00	**A. Hachemi:** Application of Direct Methods to large-scale problems
coffee break	
16:15 - 16:45	*Discussion:* Possibilities for common European projects and proposals

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5 Abstracts of talks

Do 9:00

Direct Methods for the Life Assessment of Structures at High Temperatures and Other Problems

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The presentation summaries the development of direct methods for the range of analyses that form the basis of the UK Life Assessment Method R5, originally developed by the Central Electricity Board of Great Britain and now owned by British Energy.

R5 consists of a sequence of tests that measure the proximity of increasingly complex failure modes to assumed loading histories and expected life times. These test procedures each rely upon either a direct calculation, often done by hand, or tables of known solutions. Concerns had arisen that some of the tests could produced, in certain circumstances, excessively conservative predictions, partly due to uncertainties over material data, but more significantly, with the conservative analysis methods.

Concepts of shakedown and reference stress methods diffuse the test sequence and the object of our project was to replace all stages with suitable direct methods. There was also the considerable constraint that it was unlikely that specially produced software would be maintainable.

A complete implementation of direct methods has been achieved and is now being introduced to R5 users. The novelty of the work has arisen through both theoretical and computational development. As high temperature material behaviour and non-shakedown behaviour was relevant at lower temperatures, it was necessary to extend the characterisation of steady cyclic states to these circumstances. In addition, the computational method, termed the Linear Matching Method (LMM), has been developed from a number of ad-hoc design calculation methods used in the US and refined by Boyle and Mackenzie as the Elastic Compensation Method. The essential idea in such methods is to iterate to a defined state through sequence of standard linear finite element solutions where the linear moduli are allowed to vary spatially. Hence at every iteration of a convergent process, equilibrium and compatibility are satisfied but there is some (reducing) error in the material description. This can be achieved in a number of ways. The LMM method allows strictly convergent upper bound solutions.

The presentation will summarise the basis of these methods for limit analysis, shakedown and ratchet limits. Extensions to creep behaviour are described and a comparison between the new R5 method and current methods are discussed. These methods are then applied to rolling contact problems for geotechnical problems and the behaviour of Metal Matrix Composites to varying temperature.

A list of references will be distributed at the workshop.

Do 10:00

Generalized limit analysis for poroplastic structures

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Limit and shakedown analysis in plasticity and poroplasticity are based on the hypotheses of small displacements, unlimited material ductility, Druckerian material models. In many engineering situations these hypotheses are violated, so that the above ”direct methods” employing mathematical programming concepts and algorithms are no longer valid. In such cases, safety assessment can be performed by conventional step-by-step analysis techniques. The so-called ”generalized limit analysis”, resting on the assumption of holonomic material behavior, and on a combination of limit and deformation analysis, has been developed and numerically tested with reference to elastic-plastic, possibly non-associative, materials. It is worth noting that the assumption of a holonomic constitutive law is acceptable when the loads are monotonically increasing.

This communication presents an extension of the theory and solution procedure of ”generalized limit analysis”, an extension with reference to materials, the behaviour of which may be described by means of a poroplastic constitutive law with full saturation. In particular, it will be shown that the holonomic elastic-poroplastic problem can be formulated in an analogous way as the generalized limit analysis in plasticity by introducing the pressure field p_{ss}, computed by means of a steady-state analysis, and the consequent stress field $\boldsymbol{\sigma}_{ss}$. By adopting Terzaghi “effective stress” concept, the piece-wise linearized yield function can be written as follows:

$$\boldsymbol{\varphi} = \boldsymbol{N}^T (\boldsymbol{\sigma} + \boldsymbol{m}\, p_{ss}) - \boldsymbol{H}\,\boldsymbol{\lambda} - \boldsymbol{Y} \leq \boldsymbol{0}$$

where $\boldsymbol{N}$ collects the normal vectors to yield planes, vector $\boldsymbol{m}$ is such that the pressure does not influence the shear stresses, $\boldsymbol{H}$ is the hardening matrix, $\boldsymbol{Y}$ gathers the initial yield stresses, $\boldsymbol{\lambda}$ is the vector of plastic multipliers, such as (in the holonomy hypothesis):

$$\boldsymbol{\lambda} \geq \boldsymbol{0}, \quad \boldsymbol{\varphi}^T \boldsymbol{\lambda} = 0$$

The safety load multiplier with respect to plastic collapse, local fracture because of ductility exhaustion or excessive deformation, is obtained as the maximum of the objective function in a mathematical program with complementarity constraint. This kind of problem is often referred to as MPEC (Mathematical Program under Equilibrium Constraint).

Different algorithms proposed in recent years for the solution of this problem are presented and compared in this communication, with reference to real-life problems.

Do 11:00

Limit analysis and optimization : yield function of porous frictional materials and decomposition methods

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In the first part of this talk, the macroscopic criterion of a porous material is investigated using the Gurson model of a hollow sphere, but where the matrix obeys a pressure-dependent criterion. In the second part, on the basis of a mixed rigorous kinematical approach, is presented a specific decomposition method to solve very large Limit Analysis problems. First results of its extension to the statical approach are given afterwards.

Gurson model and porous frictional materials

Thanks to the isotropy of the resultant macroscopic material, the problem is analyzed under axisymmetry assumption. Hence the model is submitted to an average strain rate on its outer boundary, and the Coulomb criterion is written as three conic inequalities, with no preliminary regularization.

As regards the finite element statical approach, a specific quadratic version was elaborated, where the stress field in the triangular elements is formulated in cylindrical coordinates. Similarly, in the kinematical approach, the displacement velocity field was chosen as specifically quadratic in the triangular finite element. To ensure the admissibility of the solution field, the velocity jumps were assigned to vary linearly on the discontinuities between the elements. To improve efficiency—essentially when there is compression—an analytical solution, derived from the solution to the problem of a cavity expanded or compressed under internal pressure [1], was superimposed on the numerical fields in both programs.

The final problems result in conic optimization, or linear programming problems after linearizing the criterion. Indeed, a fine iterative postanalysis strictly restores the admissibility of the solution in both methods, which were tested on classical axisymetric problems, and then applied so as to determine the "porous Coulomb" criterion, as was done for the "porous von Mises" criterion in [2].

For purposes of comparison a translated ”modified Cam-Clay criterion” was defined, which showed that this criterion might be considered as a satisfactory approximation for not to high values of internal friction angles, except that it does not account for the corner of the criterion on the isotropic compressive axis, unlike the original Cam-Clay criterion. Finally, a comparison with the “porous Drucker-Prager" case is presented.

Decomposition approach of LA methods

In Limit Analysis, closely bounding the limit loads generates such large numerical problems that they are often difficult to solve directly. Hence arises the idea of decomposing the original problem into smaller ones, that are as independent from one another as possible. To our knowledge, the present decomposition method, with its subproblem coordinating process, has never been proposed to solve non linear optimisation problems. All problems here have been assumed in plane strain.

Hence we used the decomposition technique detailed in [4], based on an improved version of the convex optimization method described in [3] and on breaking down meshed domain into contiguous subdomains. Each interface between subdomains is given a loading parameter series, whose associated kinematic parameters are modified at each iteration by solving an auxiliary problem for each interface.

The method was first developed for continuous piecewise linear velocity fields on the classical problem of a compressed Tresca/von Mises bar. Its discontinuous quadratic version is applied here to the von Mises/Tresca vertical slope problem, for which the best kinematic solution was $\gamma H/c = 3.782$, the best statical solution remaining 3.772.

The square mesh is composed of $N * N$ rectangles of four triangles. From N=16 to 96, the resolution is direct; from N=100 to 120, the problem is divided into two subproblems. For N=144, 160, 176, both subproblems are also split into two; the final value, strictly postanalyzed, is $\gamma H/c = 3.7784$. Note that this case, if it could be solved directly, would involve a total of 2,232,000 unknowns, 1,487,000 linear conditions and 744,000 non-linear conditions! It is also worth noting that the conic code Mosek does not converge to an optimal solution beyond N=50 for this problem. On the contrary, the optimizer [3] showed an unwavering robustness for all the cases treated. Moreover, the admissibility conditions were verified with a level of precision that was consistently better than 10^{-7}, with CPU times close to those obtained with commercial specialized codes—when these codes can be used.

Finally, an extension to the static approach is presented, and preliminary results on the problems of the compressed bar and a rigid footing on a half-plane are given.

References

[1] P. Chadwick. The quasi-static expansion of a spherical cavity in metals and ideal soils. *The Quarterly Journal of Mechanics and Applied Mathematics*; **12**: pp 52–71, 1959.

[2] M. Trillat, J. Pastor. Limit analysis and Gurson's model. *Eur. J. Mechanics A/ Solids*; **24**: pp 800–819, 2005.

[3] F. Pastor, E. Loute. Solving Limit Analysis Problems : An Interior Point Method. *Comm. Num. Meth. Engng*; **21**: 11, pp 631–642, 2005.

[4] F. Pastor. Résolution par des méthodes de point intérieur de problèmes de programmation convexe posés par l'analyse limite. *Doctoral Thesis*, University of Namur, Belgique, 2007.

Do 11:45

Limit Analysis and Shakedown of periodic media. Case of plates and beams.

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This work is devoted to the analysis of the mechanical strength of heterogeneous periodic media submitted to variable loads. The static approach of Melan and the homogenization theory are used to obtain the macroscopic strength domains in terms of admissible average stresses [1]. This direct method leads to constrained optimization problems on a three-dimensional unit cell. In order to treat general microstructures, the FEM is used and a specific formulation is introduced to take into account rigorously the periodicity conditions for the microscopic strains and stresses and the average relations between these microscopic and macroscopic quantities. The discretized constrained optimization problems are eventually performed with the software LANCELOT [2].

Using this method, emphasis is focused on the case of heterogeneous plates and beams, which exhibit periodic microstructures only in the two in-plane directions for the plates, and in the axis direction for the beams. In these cases, the homogenization approaches ([3],[4]) lead to obtain the effective models (Kirchhoff-Love plates and Navier-Bernoulli-Euler beams) by solving problems on a 3D unit cell. All the coupling (bending-tension-torsion) and the three dimensional effects (warping) are naturally taken into account with these approaches. In the framework of limit analysis, the classical definition [5] of the macroscopic strength domain is adapted [6].

This direct method is validated using some academic examples : homogeneous plate and homogeneous beams with several section geometries. Comparisons with results found in the litterature are investigated. Finally, the method is applied on some more complex three-dimensional periodic plates and beams.

References

[1] H. Magoariec, S. Bourgeois, O. Débordes. Elastic plastic shakedown of 3D periodic heterogeneous media : a direct numerical approach. *Int. J. Plasticity*; **20**: pp 1655–1675, 2004.

[2] A. R. Conn, N.I.M. Gould and Ph. L. Toint. A Fortran Package for Large-Scale Nonlinear Optimization (Release A). Springer-Verlag, Berlin, 1992.

[3] D. Caillerie. Thin elastic and periodic plates. *Math. Meth. in Appl. Sci.*; **6**: pp 159–191, 1984.

[4] P. Cartraud. Homogenéisation d'une poutre hétérogène selon son axe, de section et de période de dimensions comparable. *In Actes du 3ème Colloque National en Calcul des Structures*; Giens, pp 265–270, 1997.

[5] P. Suquet. Analyse limite et homogénéisation. *C.R. Acad. Sci.*; **296**: pp 1355–1358, 1983.

[6] S. Bourgeois, O. Débordes et P. Patou. Homogénéisation et plasticité de plaques minces. *Revue Européenne des Eléments Finis*; **7**: (1-3), pp 39–54, 1998.

Do 14:00

Engineering Application of Shakedown Analysis to Composite Structures Reinforced with Fibrous Metal Matrix Composites

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Fiber-reinforced copper matrix composites are considered as a promising heat sink material for the high-heat-flux components of nuclear fusion reactors. Plastic failure to be caused by fluctuating thermal stresses is a critical design concern regarding the structural reliability of the composite components. Shakedown analysis can provide an effective prediction of the plastic failure risk of the composite part. In this presentation the plastic failure risk of water-cooled fusion reactor components reinforced with fiber composites is treated in terms of the shakedown behavior of the composite.

At first, shakedown boundaries of two kinds of fiber-reinforced copper matrix composites were computed using both the FEM-based direct method and the incremental method (step-by-step method). Three dimensional composite unit cells were used to model the unidirectional lamina and the cross-ply laminate. Two typical bi-axial loading conditions were considered. The predicted shakedown boundaries showed a good agreement between the two numerical approaches. Effects of fiber volume fraction, temperature and applied hardening laws were investigated.

The meso-scale lamina stresses of the composite parts were computed by means of a micromechanical homogenization and compared with the shakedown boundaries of the composite. The composite stress states were located close to the shakedown boundaries indicating no critical danger of plastic failure.

Do 14:45

Stochastic structural analysis and optimal structural design with quadratic cost functions

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Problems from plastic analysis and optimal plastic design are based on the convex, linear or linearised yield/strength condition and the linear equilibrium equation for the stress (state) vector. In practice one has to take into account stochastic variations of the vector $a = a(\omega)$ of model parameters (e.g. yield stresses, plastic capacities, external load factors, cost factors, etc.), see e.g. [2, 3, 4]. Hence, in order to get robust optimal load factors x, robust optimal designs x, resp., the basic plastic analysis or optimal plastic design problem with random parameters has to be replaced by an appropriate deterministic substitute problem, cf. [1]. As a basic tool in the analysis and optimal design of mechanical structures under uncertainty, the state function $s^* = s^*(a, x)$ of the underlying structure is introduced [4]. Depending on the survival conditions of plasticity theory [2,3], by means of the state function the survival/failure of the structure can be described by the condition $s^* \leq (>)0$. Interpreting the state function s^* as a certain cost function, several relations to other cost functions, especially quadratic cost functions, are shown. Bounds for the probability of survival ps are obtained then by means of the Tschebyscheff inequality.

In order to obtain robust optimal decisions x^*, i.e., maximum load factors, optimal designs insensitive with respect to variations of the model parameters a, a direct approach is proposed then based on the primary costs (weight, volume, costs of construction, costs for missing carrying capacity, etc.) and the recourse costs (e.g. costs for repair, compensation for weakness within the structure, damage, failure, etc.), where the above mentioned quadratic cost criterion is used. The minimum recourse costs can be determined then by solving an optimisation problem having a quadratic objective function and linear constraints For each vector $a = a(\omega)$ of model parameters and each design vector x one obtains then an explicit representation of the "bes" internal load distribution F^*. Moreover, also the expected recourse costs can be determined explicitly. The expected recourse function may be represented by means of a "generalised stiffness matrix". Hence, corresponding to an elastic approach, the expected recourse function can be interpreted here as a "generalised expected compliance function", which depends on a generalised ”stiffness matrix”. Based on the minimisation of the expected primary costs subject to constraints for the expected recourse costs (”generalised compliance”) or the minimisation of the expected total primary and recourse costs, explicit finite dimensional parameter optimisation problems result – as deterministic substitute problems – for finding robust optimal design x^*, a maximal load factor, respectively. The analytical properties of the resulting programming problem are discussed, and applications, such as limit load/shakedown analysis, are considered.

References

[1] K. Marti. Stochastic Optimization Methods; Springer-Verlag; Berlin-Heidelberg-New York, 2005.

[2] K. Marti, I. Kaymaz. Reliability analysis for elasto-plastic mechanical structures under stochastic uncertainty. *ZAMM*; **86**: 5, pp 358-–384, 2006.

[3] I. Kaymaz, K. Marti. Reliability-based design optimization for elasto-plastic mechanical structures. *Computer & Structures (CAS)*; **85**: 10, pp 615–625, 2007.

[4] K. Marti. Reliability Analysis of Technical Systems/Structures by means of Polyhedral Approximation of the Safe/Unsafe Domain. *GAMM Mitteilungen*; **30**: 2, pp 211–254 2007.

Do 15:45

Reliability analysis of inelastic shell structures under variable loads

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This paper concerns the application of a new algorithm of probabilistic limit and shakedown analysis for shell structures, in which the loading and strength of the material as well as the thickness of the shell are considered as random variables. The procedure involves a deterministic limit and shakedown analysis for each probabilistic iteration, which is based on the kinematical approach and the use of the re-parameterized exact Ilyushin yield surface proposed by Burgoyne and Brennan. By imposing a consistent relationship between the velocity fields and the kinematically admissible strain rate fields, the shakedown or limit analysis problems can be reduced to a non-linear minimization problem and solved by Newton's method in conjunction with a penalty method and the Lagrange dual method. A special "smooth regularization method" was also used for overcoming the non-differentiability of the objective function. The actual Newton directions are updated at each iteration by solving a purely-elastic-like system of linear equations which ensures automatically the kinematical condition of the displacements.

The limit state function separating the safe and failure regions is defined directly as the difference between the obtained limit load factor and the current load factor. Different kinds of distribution of basic variables are taken into consideration and performed with First and Second Order Reliability Methods (FORM/SORM) for calculation of the failure probability of the structure. A non-linear optimization was implemented, which is based on the Sequential Quadratic Programming for finding the design point. Non-linear sensitivity analyses are also performed for computing the Jacobian and the Hessian of the limit state function.

The advantage of the method is that the failure under cyclic loading is treated as a time-invariant problem. Moreover, sensitivity analyses are obtained directly from a mathematical optimization with no extra computational cost. Examples were tested against literature with analytical method and with a numerical method using volume elements. The proposed method appears to be capable of identifying good estimates of the failure probability, even in the case of very small probabilities. If the limit state function is linear with the limit and shakedown analysis, then the use of FORM is sufficient and SORM is only necessary if the linearity is lost after the transformation into the standard normal space.

Reliability analysis is highly sensitive to errors in the calculation of the limit state function. Some published limit and shakedown analyses showed serious convergence problems. In order to understand the causes of such problems, the authors invite colleagues to participate in benchmark analyses of some critical test cases. Researchers may contact *m.staat@fh-aachen.de* if they wish to participate in this activity.

References

[1] Thanh Ngọc Trần, M. Staat, R. Kreißig. Probabilistic limit and shakedown analysis of thin shells using the exact Ilyushin yield surface. *Structural Safety*, 2007, submitted.

[2] M. Staat, M. Heitzer. Probabilistic limit and shakedown problems. In M. Staat, M. Heitzer (Eds.): *Numerical Methods for Limit and Shakedown Analysis. Deterministic and Probabilistic Approach.* NIC Series Vol. 15, John von Neumann Institute for Computing, Jülich, pp 217–268, 2003 http://www.fz-juelich.de/nic-series/volume15/nic-series-band15.pdf

[3] M. Heitzer, M. Staat. Limit and Shakedown Analysis with Uncertain Data. In: K. Marti (Ed.) *Stochastic Optimization Techniques, Numerical Methods and Technical Applications*. Lecture Notes in Economics and Mathematical Systems; **513**, Springer, Heidelberg, pp 241-254, 2002.

[4] M. Heitzer, M. Staat: Reliability Analysis of Elasto-Plastic Structures under Variable Loads. In G. Maier, D. Weichert (Eds.): *Inelastic Analysis of Structures under Variable Loads: Theory and Engineering Applications*. Kluwer, Academic Press, Dordrecht, pp 269–288, 2000.

Do 16:30

Shakedown analysis of problems with non associated constitutive laws and with elastic coefficients depending on the temperature

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The classical shakedown analysis is a rigorously founded theory originally proposed by Bleich, Melan and Koiter. Nevertheless, it is based on hypotheses that are often not realistic in many industrial problems. In this presentation, we discuss two possible extensions of the theory, the first one when the constitutive law is not associated, and the second one when the elastic coefficients strongly depend on the temperature.

In the first part, we recall the key ideas of the bipotential approach, a nice modelization of the non associated constitutive laws based on a possible generalization of Fenchel's inequality. It allows the recovery of the flow rule normality in a weak form of an implicit relation. This defines the class of implicit standard materials. For such behaviours, the use of variational tools leads to a weak extension of the classical bound theorems of the shakedown analysis.

Our attention is focussed on the elastoplastic materials with the nonlinear kinematic hardening rule, a model which represents accurately the cyclic plasticity of metals but is not associated. In particular, we present a reference problem for which we compare the analytical solution obtained by the proposed shakedown approach and step-by-step computations. A reliable criterion to stop the computations is proposed. Moreover, the method allows uncovering an additional "2 cycle shakedown curve" that could be useful for the shakedown design of structure. Also we compare the results for the unlimited, limited linear and non linear kinematic hardening. Finally, finite element solutions are presented for shell structures.

The second part is devoted to the influence of the temperature on the material properties. In many industrial domains, for instance in boilers of nuclear power plants or in airplane and automotive motors, the structural elements are subjected to thermal cycles of large amplitude in such way that the dependence of the elastic coefficients with respect to the temperature cannot be neglected. As far as we know, the only static shakedown theorem was formulated by J.A. König in 1969. In the light of numerical solutions, the statement of his theorem is discussed and a new shakedown condition is suggested.

Fr 9:00

Limit analysis of orthotropic laminates by the Linear Matching Method

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A limit analysis approach for orthotropic laminates, in plane stress conditions, is presented. The validity of such a theory was initially confined to structures made of standard perfectly plastic materials, nevertheless a number of researcher's efforts have provided the extension of the limit analysis theory in the context of non standard materials and/or outside the domain of perfect plasticity (e.g. Radenkovic 1961; Del Piero 1998). This has actually laid the basis for the application of the limit analysis approach to structures made of non standard materials, like soils, masonries or composites, allowing the definition of their bearing capacity. In this context can be framed the present work which considers orthotropic composite laminates under plane stress conditions. The presented approach is qualifiable as an extension, in the context of orthotropic materials, of a method known in the literature as Linear Matching Method (see e.g. Mackenzie and Boyle 1993; Mackenzie et al. 1993, 1994; Ponter and Carter 1997; Ponter et al. 2000). The LMM is a programming technique involving an iterative FE-based numerical procedure which performs a sequence of linear analyses on the structure made, by hypothesis, of a linear viscous fictitious material with spatially varying moduli. At each iteration an adjustment of the fictitious moduli is carried out so that the computed fictitious stresses are brought on the yield surface at a fixed strain rate distribution. This allows one to define a collapse mechanism, the related stresses at yield and, consequently, an upper bound to the collapse load multiplier. In the present context the LMM utilizes a fictitious linear viscous material which is orthotropic and subjected to a distribution of imposed initial stresses. The convergence of the whole procedure is guaranteed by a sufficient condition given in (Ponter et al. 2000) for a general class of yield conditions.

The key ideas of the presented procedure are: i) to apply the LMM to a second order tensor polynomial form of the Tsai-Wu failure criterion (Capsoni et al., 2001; Tsai and Wu, 1971), the latter assumed as yield surface; ii) to define the fictitious linear viscous material in such a way that the number of fictitious parameters to be updated is strongly reduced (Pisano and Fuschi, 2007). Some numerical examples are chosen both to compare the present approach with others of the relevant literature and to inquire into the effectiveness of the whole procedure to predict experimental test results for composite plates.

References

[1] D. Radenkovic. Théorèmes limites pour un materiau de Coulomb dilatation non standardisée. *Comptes Rendus de l'Académie des Sciences Paris*; **252**: pp 4103-4104, 1961.

[2] G. Del Piero. Limit analysis and no-tension materials. *International Journal of Plasticity*; **14**: pp 259-271, 1998.

[3] D. Mackenzie, J.T. Boyle. A method of estimating limit loads by iterative elastic analysis. Parts I, II, III. *International Journal of Pressure Vessels and Piping*; **53**: pp 77–142, 1993.

[4] D. Mackenzie, C. Nadarajah, J. Shi, J.T. Boyle. Simple bounds on limit loads by elastic finite element analysis. *ASME Journal of Pressure Vessels Technology*; **115**: pp 27–31, 1993.

[5] D. Mackenzie, J. Shi, J.T. Boyle. Finite element modelling for limit analysis using the elastic compensation method. *Computers and Structures*; **51**: pp 403–410, 1994.

[6] A.R.S. Ponter, K.F. Carter. Limit state solutions, based upon linear elastic solutions with spatially varying elastic modulus. *Computer Methods in Applied Mechanics and Engineering*; **140**: pp 237–258, 1997.

[7] A.R.S. Ponter, P. Fuschi, M. Engelhardt. Limit analysis for a general class of yield conditions. *European Journal of Mechanics – A/Solids*; **19**: pp 401–421, 2000.

[8] A. Capsoni, L. Corradi, P. Vena. Limit analysis of anisotropic structures based on the kinematic theorem. *Int. J. of Plasticity*; **17**: pp 1531–1549, 2001.

[9] S.W. Tsai, E.M. Wu. A general theory of strength for anisotropic materials. *Journal of Composite Materials*; **5**: pp 58–80, 1971.

[10] A.A. Pisano, P. Fuschi. A numerical approach for limit analysis of orthotropic composite laminates. *Int. J. for Num. Meth. in Eng*; **70**: pp 71–93, 2007.

Fr 9:45

Fatigue and shakedown concepts

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The fatigue of materials and structures is the consequence of plastic and damage mechanisms at the grain scale, under cyclic loadings. These mechanisms lead to a dissipation at this scale which induces an increase of the temperature. Luong [4] proposed a thermodynamical framework which allows to establish a link between the observed dissipative regime and the fatigue limit of many materials. Some recent works extend Luong's observations and underline the links between the fatigue phenomenon, the thermoplasticity and the dissipation [1]. This dissipative framework seems to allow the construction of a unified approach in fatigue, based on shakedown concepts [2]. However, a purely thermoplastic dissipative framework stumbles on the mean stress effect in fatigue. Indeed, the application of a mean stress under cyclic loading leads generally to a variation of the S-N curve and, consequently, of the macroscopic fatigue limit. In the same way, the observed macroscopic cyclic temperature variations depend also on mean stress. Mean stress is generally taken into account in the multiaxial fatigue criteria with a postulated linear dependence of the shear component on the hydrostatic pressure; this is the case of the Dang Van criterion [3]. As classical purely plastic models depends generally only on deviatoric part of stress tensor, dissipation is however independent on the hydrostatic part.

In order to represent all these experimental observations, a new plasticity-damage approach in fatigue, provided by the recent work of Monchiet et al. [5] is proposed. The main feature consists in the incorporation of some observed damage mechanisms (reported in literature for materials involving faced-centered-cubic structures) in the multiscale framework proposed initially by Dang Van. The originality of the study lies in the consideration of the scale of Persistent Slips Bands (PSB) at which appear the damage micromechanisms. Actually, the PSB constitute preferential sites for fatigue cracks nucleation. This approach is derived in the Thermomechanics of Irreversible Processes framework and allows to define dissipative terms associated to damage and plasticity.

References

[1] E. Charkaluk, A. Constantinescu. Dissipation and fatigue damage. *MP Materialprüfung*; **46**: 10, pp 524–530, 2004.

[2] A. Constantinescu, K. Dang Van, H. Maïtournam. A unified approach for low and high cycle fatigue based on shakedown concepts. *Fat. Frac. Engng. Mat. Struct.*; **26**: 6, pp 561–568, 2003.

[3] K. Dang Van. High-Cycle fatigue in the context of mechanical design. *CISM courses and lectures n. 392*; editors: K. Dang-Van and I.V. Papadopoulos; Springer-Verlag; **46**: 10, pp 1691–1710, 1999.

[4] M. P. Luong. Infrared thermographic scanning of fatigue in metals. *Nucl. Engng. and Design*; **58**: pp 363–376, 1995.

[5] V. Monchiet and E. Charkaluk, D. Kondo. A plasticity-damage based micromechanical modelling in high cycle fatigue. *C.R. Mécanique*; **334**: 2, pp 129–136, 2006.

Fr 10:45

Cohesive Zone Model within the Finite Element Framework Implementations, Enhancements, Applications

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Introduction

Advanced development of the finite element (FE) method to take discrete fracture and failure modes into account is a current field of research. Cohesive elements, based on the Dugdale Barenblatt crack tip model, are introduced as a common formulation within this context. Such cohesive surfaces are applied to introduce a strong displacement discontinuity between standard finite elements to represent the opening faces of fracture and delamination processes. An appropriate material formulation, in terms of a traction separation law (TSL), depicts both the maximum traction at failure initiation and the necessary energy to form new free surfaces. The first part of the presentation is dedicated to general aspects of cohesive element and material formulation. Furthermore, characteristics and considerations of application within a FE framework are pointed out.

Time dependent cohesive formulation

A wide range of polymeric materials show time dependent material characteristics, e.g. loading rate dependencies, relaxation and hysteresis effects. A viscoelastic material formulation provides these phenomena for numerical simulations. It can be shown, that fracture and delamination events of such materials also depict such a behaviour. A viscoelastic traction separation law on the basis of a rheological model is introduced to assign these characteristics to the fracture process zone of polymeric materials. Besides some aspects of the viscoelastic formulation, an alternative approach, based on a pure rate dependent TSL, is presented. By means of some numerical examples, both models are compared for different dynamic loading schemes. Furthermore, the introduced formulation is applied for the simulation of an experimentally investigated peel film system of Polyethylene/Polybutene-1 foils.

Model adaptive implementation

The consideration of cohesive surfaces within the initial finite element model induces some relevant difficulties. Especially for a large number of such elements and for computations without definite specified crack paths, these drawbacks become relevant. Here, the number of provided unknowns and softening effects due to the initially elastic branch of the traction separation law are exemplary emphasised. To overcome these drawbacks, a model adaptive implementation in dependence of the present crack propagation is proposed. Beneath the fully uninfluenced solution up to first crack initiation, the proposed approach provides an optimum mesh in dependence of the current crack state. In contrast to already existing formulations of that type, the focus of the present implementation is on failure processes that can be described at quasi static conditions within an implicit finite element framework. The general procedure of such an adaptive implementation as well as particular aspects of related cohesive material and element formulation are given within this contribution. A numerical investigation of a three point bending beam, illustrates the differences between the proposed cohesive implementations.

Fr 11:30

Tribology and mechanical systems functionalities

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Predicting life of complex mechanical systems is a challenging problem. At design stage, or for expertise purposes, a strong need for specific tools, taking into account more and more phenomena in a short calculation time, emerged. Contact often appear as key points in this field. In a rocket or car engine, hundreds of contacts can be numbered, among them, bearings, seals, welded metal sheets; and failure is mostly occurring around these contacts.

In our lab, work is focused on contact from experimental and modelling points of view. Various fields are investigated:

- Rolling sliding contact fatigue, fretting fatigue and fretting wear: a direct method for shakedown analysis allows an easy determination of shakedown status (elastic or plastic) and in the elastic shakedown case, of residual stresses effect on life of the structure.
- Microgeometry of surfaces (roughness, surface waves,...):
  - an original stochastic model for lubrication takes into account transitory phenomena encountered when starting the engines, pumps, or..; if seizure occurs in these early stages, the whole life of the engine can be drastically reduced and the stationary cycle which is the starting point for direct methods or limit analysis cannot be reached.
  - Starting form stochastic description of microgeometry and a simplified elastoplastic model for a single coated asperity a stochastic estimation of gas leak through metallic softly coated seals have been derived.
- Crash tests on welded metal sheets and optimisation of steel structure according to mechanical needs.
- Environmental aspects with studies on hydrogen effect on metal sheet properties, and qualification of controlled ecotoxicologic lubricants or materials for biomechanics.

Fr 12:15

Extended Melan's theorem for the study of frictional systems

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Shakedown (SD) of frictional systems is important in many applications ranging from technological areas such as frictional dampers, contact joints in mechanical and biomechanical assemblies subjected to fretting.

The evident analogy between simple macroscopic Coulomb friction and plastic perfect plasticity for a uniaxial specimen, has often attracted scientists, who have used plasticity theories in computational techniques also for friction, and have speculated whether shakedown theories may apply to frictional systems. Drucker already in the 1950's warned that the analogy is not strong, since friction is non-associative. In recent papers (IJSS, CR-Mech) from Anders Klabring, Jim Barber, and myself, conditions were obtained for restricted versions of Melan's theorem to apply for Coulomb frictional systems for which we know a priori there is always contact (or we can check independently), i.e. in practice only for special elastically uncoupled problems where separation is avoided by taking constant the normal loads, and either complete or receding contacts.

Here, we generalize these results for the case of separation and cyclic slip. Hence, it will be recognized that we can understand the response of these systems to cyclic loadings, independently on the initial transient.

An example is given for the standard Hertzian contact for which Mindlin's solution applies and generally if unloading involves also the normal load, there would be release of residual stresses associated with the loss of contact from the edges of the contact area. We suggest hence that the energy dissipation in the case of cyclic slip is independent on the initial transient, and could be used to devise a numerical variational formulation to solve the contact problem.

Fr 14:30

Force method based procedures in the limit equilibrium analysis of 2D & 3D framed structures

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It is a well known fact that in order to estimate the strength of structures under monotonically increasing loading, or under cyclic loading, one could avoid detailed elastoplastic analysis and resort, instead, to direct limit or shakedown analyses, based on mathematical programming. They may be both referred to as limit equilibrium analyses. It is obvious that these analyses may be better developed within the framework of an equilibrium based method like the force method. Nevertheless, the displacement method has been used almost exclusively, since it has a better potential for automation. In the case of framed structures, in particular, the difficulty for such an automation of the force method is the way to select their redundancy.

The graph representation of a frame may provide such automation, in which, a member and a node of the frame is a member and a node of a directed graph, respectively. The foundation node may be considered as an extra node and each foundation node is connected to the ground node with an extra member; then there is a unique number of independent closed loops. This number is called Betti's number and is given by $M - N + 1$, where M, N is the total number of the members and the nodes of the graph, respectively.

An algorithm to identify such an independent set of loops has been proposed and has been applied to the design of plane frames [1]. It is quite easy to implement and the only knowledge from graph theory required is a shortest path technique between two points of a 2D graph. A statical basis may then be provided since each cycle is three times statically indeterminate. In a shakedown analysis, this basis may also be used to formulate the flexibility matrix and thus establish the elastic solution [2]. The way the basis is found makes possible to establish this matrix in a compact skyline form, like the one used for the stiffness matrix in a displacement based method. Therefore, any known decomposition methods may be used to solve the system of compatibility equations. This algorithm has recently been used in the case of 3D frames, as its graph representation is equivalent to an admissible embedding into a 2D polyhedron [3].

Equilibrium with respect to external loads is provided along cantilevers which are established using, once again, the same shortest path technique between the points of load application and the foundation nodes. Examples of application will also be given.

References

[1] K.V. Spiliopoulos. On the automation of the force method in the optimal plastic design of frames. *Comp. Meths. Appl. Mech. & Engng.*; **141**: pp 141–156, 1997.

[2] K.V. Spiliopoulos. A fully automatic force method for the optimal shakedown design of frames. *Journ. of Comp. Mech.*; **23**: 4, pp 299–307, 1999.

[3] K.V. Spiliopoulos, M. Marinou. Automatic Limit Analysis of Three Dimensional Frames. *Proc. COMPLAS 2007*; Barcelona, Spain, 2007.

Fr 15:15

Application of direct methods to large-scale problems

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There are many industries producing or operating safety-critical structures under heavy loading conditions. Many of these structures or structural components behave ductily and undergo plastic deformations under severe loading or under some normal operating conditions. Their lifetime may be determined by fatigue failure or incrementally increasing deformations due to plasticity.

A better understanding of the behaviour of such structures under complex loading conditions may considerably improve their design. In fact, the direct industrial need for end-users of a validated design and assessment method is to improve structural design.

In combination with well-known numerical tools such as the finite element method, it is in principle possible to study the behaviour of structures by performing a series of incremental elastic-plastic analyses. However, for complex loading histories, the required numerical expense of this kind of procedure may be very high. Furthermore, an accumulation of errors cannot be excluded in principle. Whereas, direct methods, namely limit and shakedown analysis, is efficient and practical approach for the prediction of the long-term behaviour of engineering structures under arbitrary cyclic loading and provide simply and rapidly information on admissible states without recurring on the evolution on deformation and material properties as functions of the loading history. The lower bound direct methods lead to a constrained problem of nonlinear mathematical programming in conjunction with finite element methods and need basically:

- the solution of the purely elastic reference problem,
- the use of an optimisation procedure, to determine the safety factor against failure and to construct a time-independent self-equilibrated residual stresses field.

The considered problem is a non-linear optimisation problem with constraints, which necessitates for the engineering applications a very large number of optimisation variables and a large amount of computer memory. In the following, we apply the interior point method to solve this large-scale problem with a reasonable computer time. The lower-bound direct methods are applied to three-dimensional structures. The obtained numerical results are compared to the existing results in literature by showing the advantages of this method.